Improved Deterministic Modeling for Safeguards Measurements

Jawad R. Moussa, Anil K. Prinja

University of New Mexico

UNM PI: Anil K. Prinja, prinja@unm.edu

Consortium for Monitoring, Technology, and Verification (MTV)

Introduction and Motivation

- Neutron fingerprinting by neutron multiplicity measurement
  - Proven technique to establish unique signatures for tight identification of nuclear fuel composition; accountability and control of nuclear material at every stage of the fuel cycle
  - Modeling and simulation: complements experiments, provides predictive capability
- Project goal is to develop hierarchy of numerical solution approaches
  - Lumped or point kinetic models for rapid scoping and estimation: Monte Carlo and deterministic
  - Full phase deterministic methods for more refined solutions → energy and angle correlations
  - Monte Carlo for highly refined solutions and benchmarking (MCNP6, PoliMi)

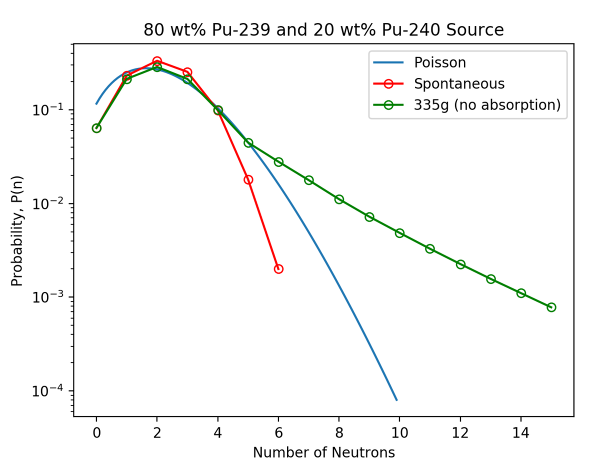
Mission Relevance

- Mitigating proliferation pathways through modeling and simulation advances to support nuclear material detection
- Expanding and making accessible to the community advances in predictive science capabilities in the nuclear security space
- Preparing future experts in nuclear security through pedagogy, defining and solving innovative research problems, and developing advanced technical skills in theory, modeling and simulation

Technical Approach & Results

Background

- Neutron Multiplicity Counting (NMC) is based on observing the statistical fluctuations of emitted neutrons as deviation from a Poisson distribution. Analysis of these deviations provides information pertaining to the fissile nuclear material of interest.



- Point Kinetic Model was considered – Spatial and Energy dependence have been ignored.
- Joint probability distribution $P_{n,m}(t)$ was defined to study the number of neutron counts.

$P_{n,m}(t)$: The probability of n neutron existing at time t with m neutrons being detected up to that time.

Forward Master Equation

- The forward master equation is formulated by conducting a probability balance of all independent mutually exclusive interaction a neutron may undergo in a system

$$
\begin{aligned}
P_{n,m}(t) = {} & (1-(S+n\lambda_T)\Delta t)P_{n,m}(t) && (No\ Event)\\
& +S\Delta t\sum_{j=0}^{J_S}\left[q_j^S P_{n-j,m}(t)\right] && (Spontanous\ Fission\ Event)\\
& +\lambda_f\Delta t\sum_{j=0}^{J_I}\left[q_j^I(n+1-j)P_{n+1-j,m}(t)\right] && (Induced\ Fission\ Event)\\
& +\lambda_c\Delta t(n+1)P_{n+1,m}(t) && (Capture\ Event)\\
& +\lambda_z\Delta t(n+1)P_{n+1,m-1}(t) && (Leakage\ Event)
\end{aligned}
$$

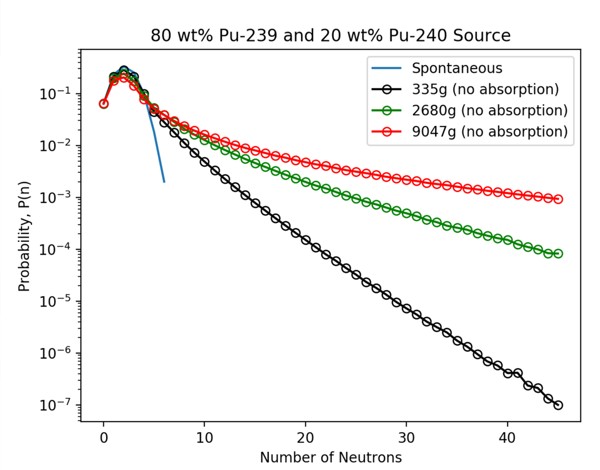
$$
\frac{d}{dt}P_{n,m}(t) = -(S+n\lambda_T)P_{n,m}(t) + S\sum_{j=0}^{J_S}\left[q_j^S P_{n-j,m}(t)\right] + \lambda_f\sum_{j=0}^{J_I}\left[q_j^I(n+1-j)P_{n+1-j,m}(t)\right] + \lambda_c(n+1)P_{n+1,m}(t) + \lambda_l(n+1)P_{n+1,m-1}(t), \quad P_{n,m}(0) = \delta_{n,0}\delta_{m,0}
$$

Stochastic Simulation Algorithm (SSA) Monte Carlo

- From the probability balance above, an algorithm is derived to simulate the state of a system.
- The state of the system at time t is defined as {n, m, f,…}

Algorithm:
  t = Sample time to event
  while ($t < t_{final}$)
    Sample which event occurred
    Change the state of the system
    t += Sample time to next event

Event	Probability	State Change
Spontaneous Fission	$Sdt\,.\,q_k$	$n \to n + k$
Induced Fission	$n\lambda_f dt.\ \mathrm{p_k}$	$n \to n + k - 1$ $f \to f + 1$
Capture	$n\lambda_c dt$	$n \to n - 1$
Leakage	$n\lambda_l dt$	$n \to n - 1$ $c \to c + 1$
No Event	$1 - (S + n\lambda_T)dt$	0



The algorithm was used to reproduce results previously published by A. Enqvist, S. Pozzi, I. Pa´zsit (2009) where the number of neutron counts produced by a single source event were simulated for three different sample masses. As expected, the tail deviates from a Poisson with increasing mass of fissile material.

Deterministic Method

- A deterministic solution for the neutron number PDF was produced
- $P_n(t)$ satisfies the Forward Master Equation:

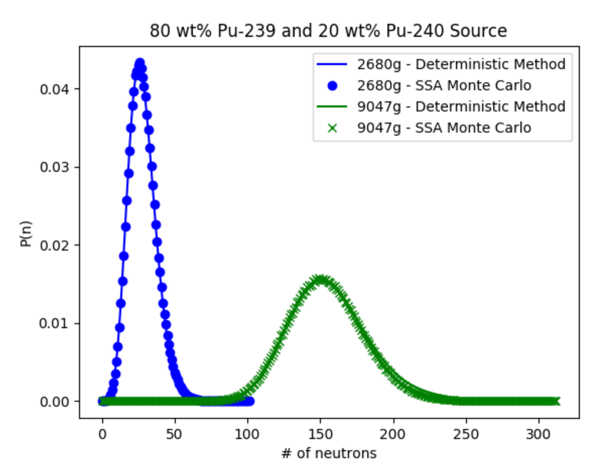
$$
\frac{d}{dt}P_n(t) = -(S+n\lambda_a)P_n(t) + S\sum_{j=0}^{J_S}\left[q_j^S P_{n-j}(t)\right] + \lambda_f\sum_{j=0}^{J_I}\left[q_j^I(n+1-j)P_{n+1-j}(t)\right] + \lambda_c(n+1)P_{n+1}(t), \quad P_n(0) = \delta_{n,0}
$$

- This constitutes an open set of lineally coupled ODEs. However, guided by the Monte Carlo simulation, the set can be truncated at N where

$$P_n(t) \approx 0, \qquad n > N$$

- The set of equation can be represented in matrix form and solved using Eigenvector decomposition.

$$\frac{d}{dt}\vec{P}(t) = \mathbf{M}\vec{P}(t); \quad \vec{P}(0) = \{1,0,\dots,0\}^T$$



Both the SSA and deterministic methods were compared against each other for runtime efficiency.

Method	Runtime (s)	
	2680g	9047g
Monte Carlo	37	121
Deterministic	0.03	1.4

Expected Impact

- Provide a deeper understanding of the statistical uncertainties of radiation signals from weak sources

MTV Impact

- Create linkages with MTV partner universities through workshop participation
- Strengthen national lab connections in nuclear security areas through joint research and internship opportunities for undergraduate and graduate students

Conclusion

- The Stochastic Simulation Algorithm (SSA) Monte Carlo: versatile, memory and performance efficient for multiplicity simulations in lumped geometry
- Deterministic methods: orders of magnitude speedup potential

Next Steps

- Simulate conditions under which overlapping chains are important
- Further expand understanding of SSA performance and capability
- Add multigroup neutron energy dependence to SSA – benchmarks for later deterministic solution
- Shift focus of deterministic approach to backward Master equation formulation for external neutron count distributions

This work was funded in-part by the Consortium for Monitoring, Technology, and Verification under Department of Energy National Nuclear Security Administration award number DE-NA0003920

National Nuclear Security Administration