MTV Student Virtual Research Symposium

Neutron Multiplicity Distributions Using Stochastic and Deterministic Solution Methods

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Introduction and Motivation

- Neutron Multiplicity Counting (NMC) relies on statistical fluctuations of emitted neutrons to identify SNM
    - Proven technique to establish unique signatures for identification of nuclear fuel composition; accountability and control of nuclear material at every stage of the fuel cycle
    - Modeling and simulation: complements experiments, provides predictive capability
- Project goal is to develop hierarchy of deterministic solution approaches, from point kinetic to higher fidelity models
- Point kinetic model was developed and solved for the probability distribution of counts, $P_n(t)$
- The Feynman-Y was computed using a unique Monte Carlo method and by deterministically solving for the moments of the count probability distribution $P_n(t)$

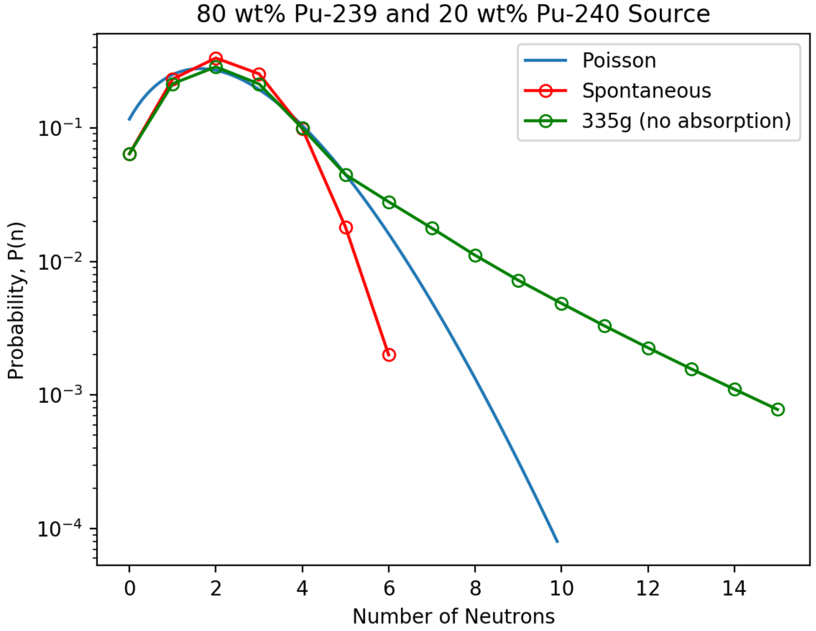

Fig.1 Illustration of the effect of neutron multiplication on the count probability distribution $P_n(t)$

Mission Relevance

- Mitigating nuclear proliferation pathways through advances in modeling and simulation to support nuclear material detection
- Expanding and making accessible to the nuclear security community advances in theory and computational modeling of neutron signatures
- Educating and training future experts in nuclear security through innovative research that develops advanced technical skills in theory, modeling and simulation

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Stochastic Simulation Algorithm (SSA) Monte Carlo

- Conduct a probability balance of all independent, mutually exclusive events in incremental time interval (Forward Master Equation)
- Algorithm developed to simulate the time evolution of the stochastic state of the system $\{n, m, f \ldots\}$
- Algorithm is more versatile, memory and performance efficient than traditional event-based Monte Carlo for point kinetic systems

Table 1. SSA Monte Carlo state changes effects by event

Event	Probability	State Change
Spontaneous Fission	$Sdt\,.\,q_k$	$n \rightarrow n + k$
Induced Fission	$n\lambda_f dt.\ \mathrm{p_k}$	$n \rightarrow n + k - 1$
Capture	$n\lambda_c dt$	$n \rightarrow n - 1$
Leakage	$n\lambda_l dt$	$n \rightarrow n - 1$ $m \rightarrow m + 1$
No Event	$1 - (S + n\lambda_T)dt$	0

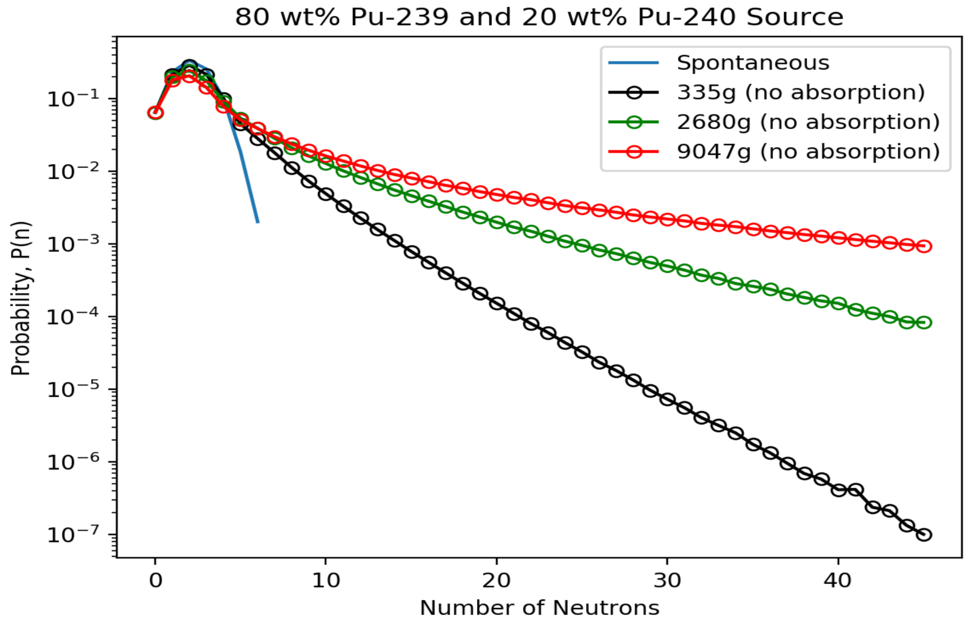


Fig.2 number of neutron counts produced by a single source event for three sample masses of 80 wt% Pu239 and 20 wt% Pu240

- SSA algorithm was used to reproduce published results: A. Enqvist, S. Pozzi, I. Pazsit (2009)

Backward Master Equation (BME) for Number of Neutron Counts

- Traditional Monte Carlo calculations have been known to require very long runtimes. On the other hand, deterministic methods provide orders of magnitude speedup potential.

- The BME is formulated by conducting a probability balance of all independent mutually exclusive events to develop equations for the probabilities $P_n(t|s)$ and $\Theta_n(t|s),\ n \geq 0$

$\boldsymbol{P_n(t|s)}$: The probability $\boldsymbol{n}$ neutrons leaking from the system by time $\boldsymbol{t}$ given that one neutron was introduced into the system at an earlier time $\boldsymbol{s}$

$\boldsymbol{\Theta_n(t|s)}$: The probability $\boldsymbol{n}$ neutrons leaking from the system by time $\boldsymbol{t}$ given that an intrinsic random source is turned on at an earlier time $\boldsymbol{s}$

BME Continued...

- Goal is the count distribution in a given detector time gate
- Two problems must be solved to achieve this goal
  - Single chain count distribution in fixed time Interval
  - Intrinsic random source count distribution in fixed time interval

- Backward Master Equation in a fixed time gate, for counts $n = 0, 1, \ldots$

Single Chain Equation:

$$-\frac{\partial}{\partial s} P_n(t|s) = -\lambda_T P_n(t|s) + \lambda_c \delta_{n,0} + \lambda_l[\delta_{n,1} I_D(s) + \delta_{n,0} I_D(s)] + \lambda_f \sum_{v=0}^{\hat{v}} p_v \sum_{|\vec{n}_v|=n} \prod_{k=1}^{v} P_{n_k}(t|s)$$

$$P_n(t|t) = \delta_{n,0}$$

Auxiliary Equation to Account for Random Intrinsic Source:

$$-\frac{\partial}{\partial s} \Theta_n(t|s) = -S\Theta_n(t|s) + S \sum_{k=0}^{K_s} q_k \sum_{\vec{n}_k + m_k = n} \prod_{j=1}^{k} P_{n_j}(t|s) \Theta_{n_k}(t|s)$$

$$\theta_n(t|t) = \delta_{n,0}$$

Generalized Binary Fission Model

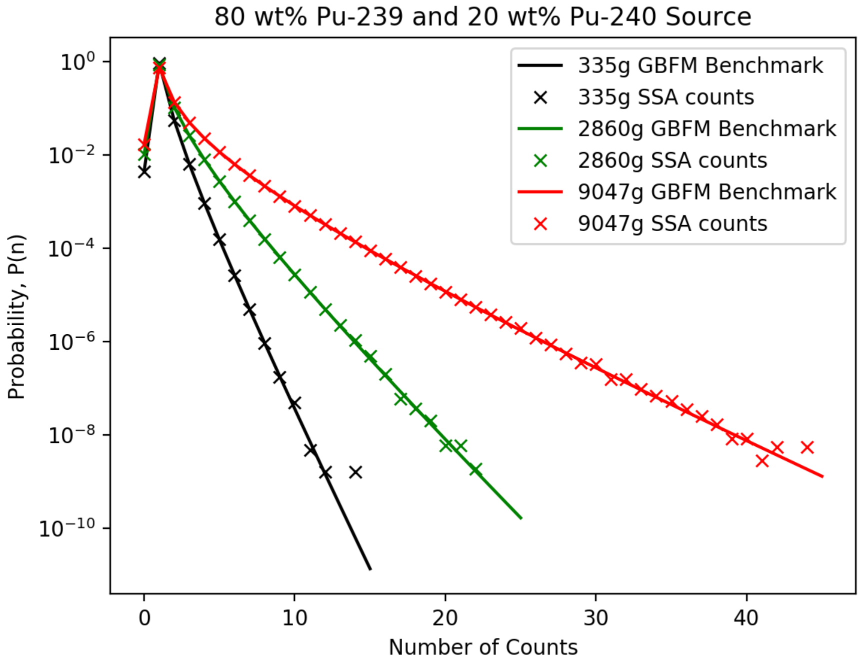
- An analytical solution for the count distribution was developed for a simplified fission multiplicity to benchmark the SSA algorithm
- The probability of a fission event producing more than 2 neutrons was take to be 0

$$p_n = 0; \qquad n > 2$$

- A steady state solution was obtained for the single chain BME

$$P_n = C_{n-1}\left(\frac{\sqrt{(1-4p_f p_c)}}{p_f}\right)\left(\frac{p_l p_f}{(1-4p_f p_c)}\right)^n$$

$$C_n = \frac{(2n)!}{n!\,(n+1)!} \qquad - \textit{Catalan numbers}$$



Generating Function Equation

"A generating function is a device somewhat similar to a bag. Instead of carrying many little objects detachedly, which could be embarrassing, we put them all in a bag, and then we have only one object to carry, the bag"

$$G(z;t|s) = \sum_{n=0}^{\infty} z^n P_n(t|s) \qquad \Leftrightarrow \qquad P_n(t|s) = \frac{1}{n!}\frac{\partial^n}{\partial z^n} G(z;t|s)\Big|_{z=0}$$

- The Generating Function transform reduces the infinite number of equations for $P_n(t|s)$ to a single pde with an auxiliary parameter z:

$$-\frac{\partial G(z;t|s)}{\partial s} = -\lambda_t G(z;t|s) + \lambda_c + \lambda_l + \lambda_f \sum_{v=0}^{\hat{v}} p_v G^v(z;t|s)\,; \qquad -\frac{\partial H(z;t|s)}{\partial s} = -SH(z;t|s) + S\left[\sum_{k=0}^{K_s} q_k G^k(z;t|s)\right] H(z;t|s)$$

- GFE aren't always solvable however, taking the derivatives and setting $z = 1$ gives the moments of $P_n(t|s)$
  - First Moment : $\frac{\partial}{\partial z} G(z:t|s)|_{z=0} = \bar{n}$ $\quad mean$
  - Second Moment: $\frac{\partial^2}{\partial z^2} G(z:t|s)|_{z=0} = \overline{n(n-1)}$ $\quad second\ factorial\ (f_2)$

Feynman-Y

- The Feynman-Y counting statistic is defined as the ratio of a count distribution's variance to its mean

$$\frac{\sigma^2}{\mu} = 1 + Y$$

- The Feynman-Y is an important quantity for identifying special nuclear materials
- It is a measure of the systems multiplication and represents the excess variance to quantify the deviation from a purely uncorrelated Poisson source
- Y increases with increasing neutron multiplication and is zero if the the distribution is purely Poisson
- At larger time gates the value of Y becomes time independent

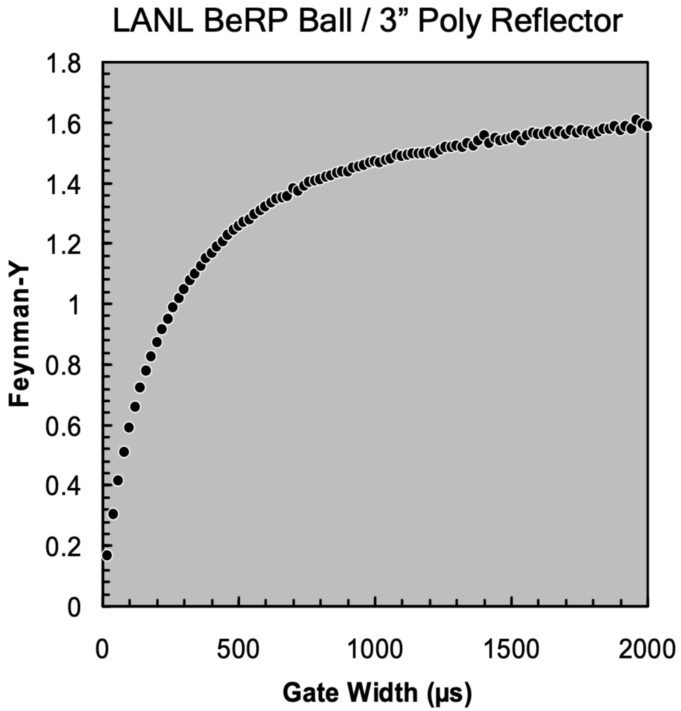
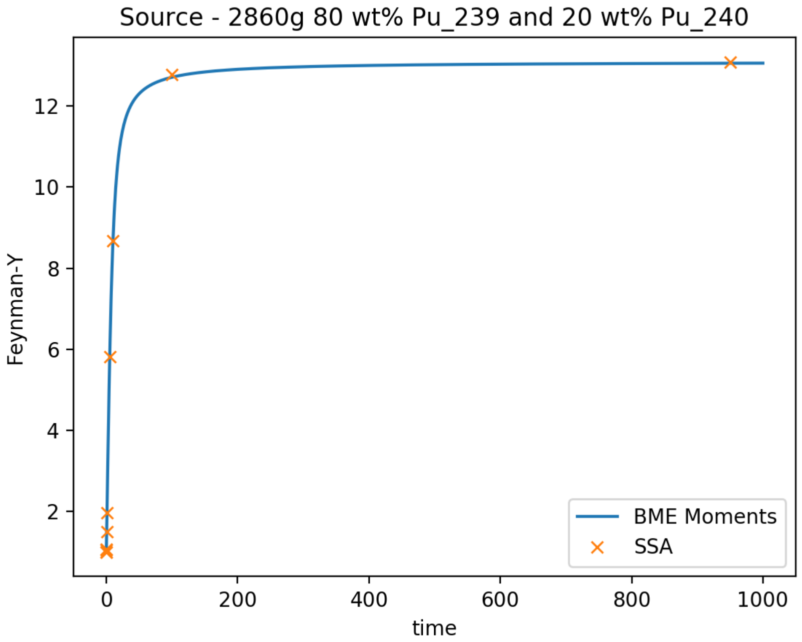


Fig. 3. The Feynman-Y statistic plotted against gate width for the LANL BeRP ball with a 3" Poly reflector

J. Mattingly, "Computation of neutron multiplicity statistics using deterministic transport," 2009 IEEE Nuclear Science Symposium Conference Record (NSS/MIC), Orlando, FL, 2009, pp. 1350-1355, doi: 10.1109/NSSMIC.2009.5402335.

Feynman-Y Continued...

- $80\ wt\%\ Pu_{239}\ and\ 20\ wt\%\ Pu_{240}$ spherical source was simulated
- Data was obtained by running a detailed transport calculation of the problem and compared with previously published work that used the same source



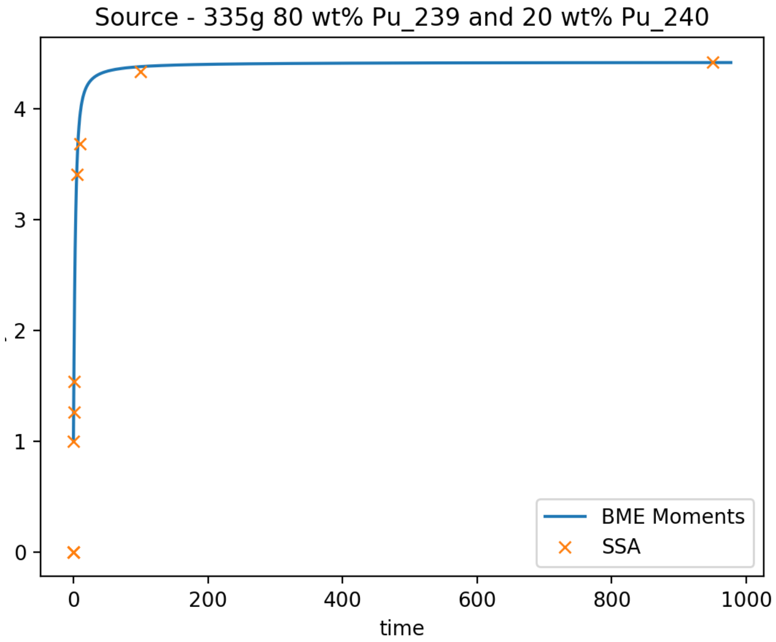
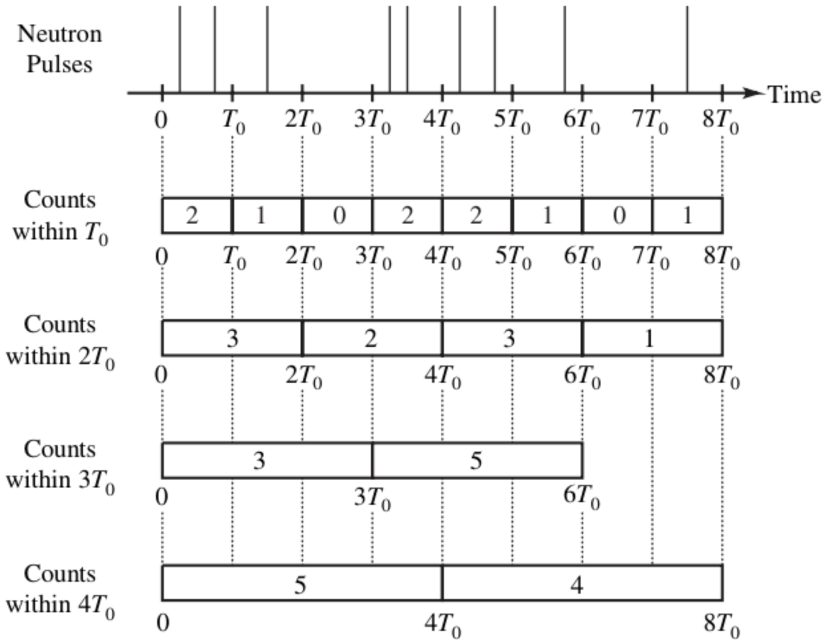


Fig.4. Comparison of the Feynman-Y for two different masses of 80 wt% Pu239 and 20 wt% Pu240 source produced but analytically solving the moments of the count probability distribution and SSA method

Experimental Approach

- Neutron multiplicity counters are operated in list mode
- A bunching method is used to analyze the data collected



Y. Kitamura, T. Misawa, A. Yamamoto, Y. Yamane, C. Ichihara, H. Nakamura, Feynman-alpha experiment with stationary multiple emission sources, Progress in Nuclear Energy, Volume 48, Issue 6, 2006, Pages 569-577

Fig.5. Illustration of the bunching method used to analyze data for the Feynman-Y method

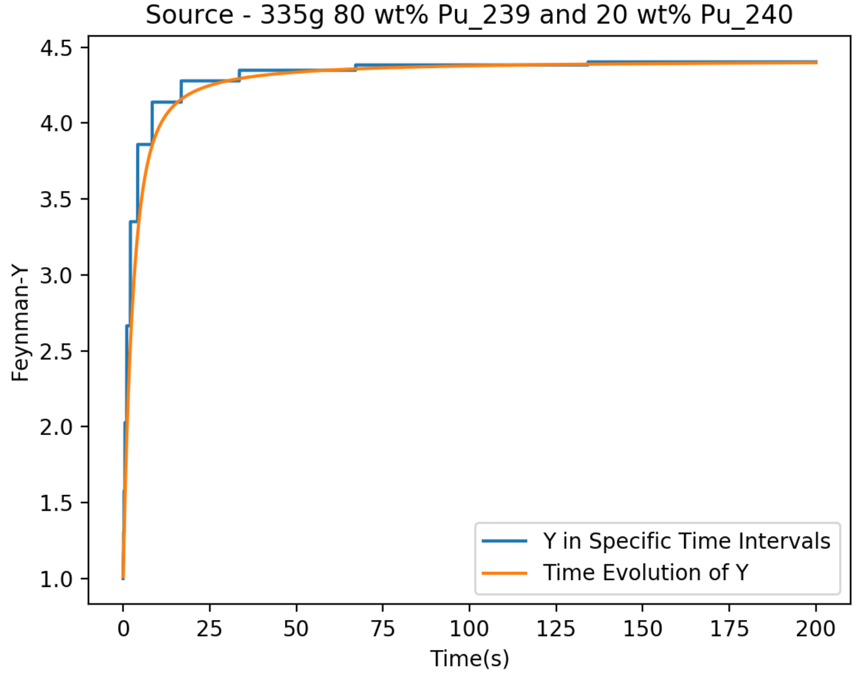


Fig.6. Comparison of the Feynman-Y for a 335g 80 wt% Pu239 and 20 wt% Pu240 source produced over a large time gate vs individual time gate increasing in size

Impact

- Expected Impact
  - Provide a deeper understanding of the statistical uncertainties of radiation signals from weak sources
  - Provide deterministic computation alternative to expensive Monte Carlo simulation of neutron signatures
- MTV Impact
  - Create linkages with MTV partner universities through workshop participation; establish collaborations
  - Strengthen national lab connections in nuclear security areas through joint research and participation in internships

Conclusion

- Stochastic Simulation Algorithm (SSA) was demonstrated to be efficient for simulating the stochastic state (neutron number, counts, fission numbers …) as a function of time for point kinetic models
- Forward and Backward Master equation formulations provide efficient deterministic options for computing count statistics such as Feynman-Y as well as count number distributions
- The time evolution of the Feynman-Y over an infinitely large time interval produces the same value are the bunching method over several time intervals with increasing gate widths
- Advanced modeling and simulation capabilities and creation of human capital in nuclear security areas support NNSA mission

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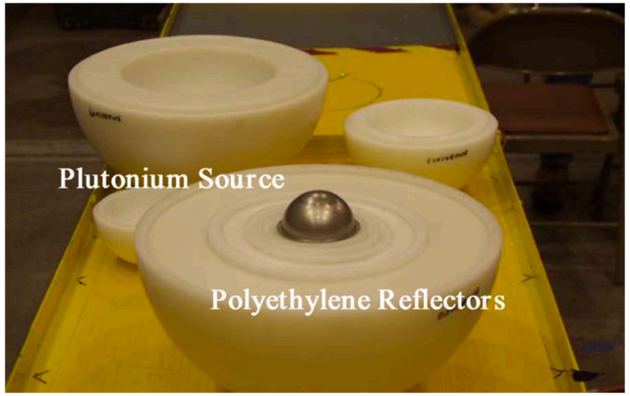
Next Steps

- Reproduce the Feynman-Y by developing a Monte Carlo code to simulate the bunching method
- Add spatial dependence and advance model to 1D spherical geometry
- Gain better understanding of how MCNPX-PoliMi and MCNP6 can be used for such measurements in more complex geometry



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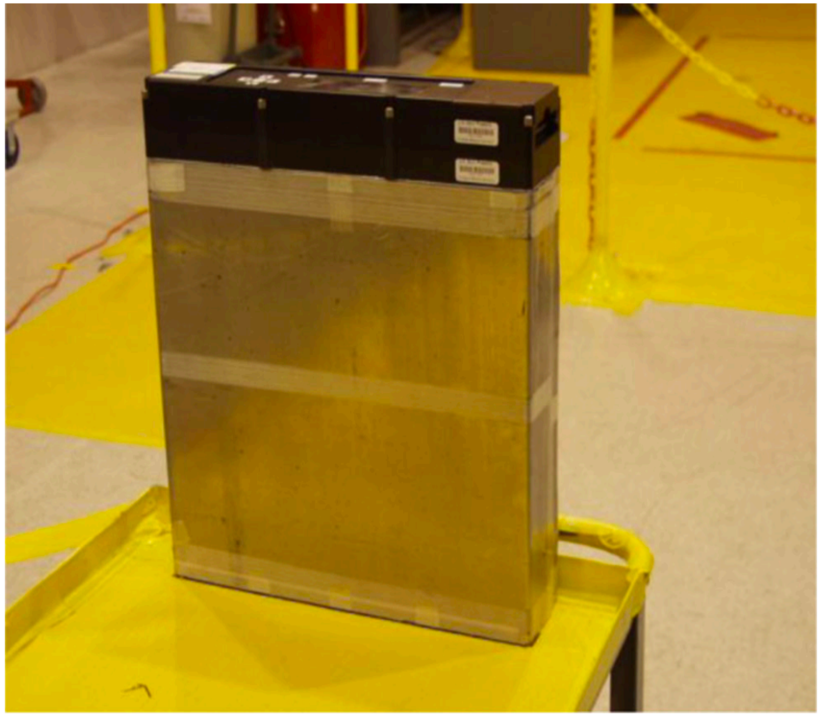

E. C. Miller, J. K. Mattingly, S. D. Clarke, C. J. Solomon, B. Dennis, A. Meldrum & S. A. Pozzi (2014) Computational Evaluation of Neutron Multiplicity Measurements of Polyethylene-Reflected Plutonium Metal, Nuclear Science and Engineering, 176:2, 167-185, DOI: 10.13182/NSE12-53

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