



# Modeling of Plutonium (Pu) Production in Foreign Nuclear Fuel Cycles (FNFC)

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# Introduction and Motivation

- Nuclear weapon states outside of the NPT and facilities not under the IAEA safeguards
  - Nuclear security (non-state actors) and safeguards (state actors) concerns of plutonium production in these states



- Motivation: Identify forensics signatures and proliferation identifiers for these Nuclear Fuel Cycles

# Mission Relevance

- Develop an enhanced and innovative FNFC monitoring method
  - **Nuclear forensics signatures and proliferation identifiers of FNFC**
    - Modeling and experimental efforts of Pu and  $^{233}\text{U}$  production reactors and associated fuel cycle facilities
    - Accurate methods for discriminating source of Pu and  $^{233}\text{U}$  (for nuclear security and safeguards applications)



# Technical Work Plan in Collaboration with National Labs

- **Task 1:** High-fidelity computational modeling of a set of FNFCs

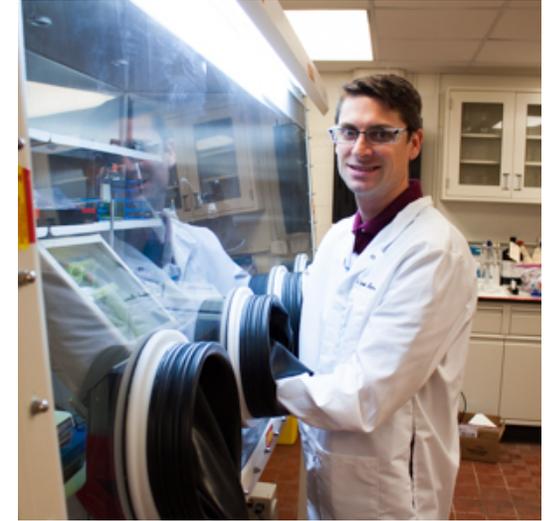
- U and Th fueled reactors (research and power) with fuel reprocessing (aqueous and electrochemical)

- **Task 2:** Experiments to point-validate the results from the modeling efforts.

- Low-fluence neutron irradiation of U and Th surrogates using neutron spectrum modifying capsules to produce milligram quantities of Pu and  $^{233}\text{U}$
- Radiochemical separations and contaminant analyses using alpha, gamma and mass spectrometry

- **Task 3:** A maximum likelihood data analytics method to discriminate source reactor-type of Pu and  $^{233}\text{U}$

- Fuel burnup and time since irradiation predictions



$$L(M|r_{mes}) \propto f(r_{mes}|M) = \prod_{j=1}^n \frac{1}{\sigma_{j, sim} \sqrt{2\pi}} \exp \left\{ -\frac{(r_{j, mes} - r_{j, sim})^2}{2\sigma_{j, sim}^2} \right\}$$

$$\text{Log } L(M|r_{mes}) = \sum_{j=1}^n \left[ \log \left( \frac{1}{\sigma_{j, sim} \sqrt{2\pi}} \right) - \frac{(r_{j, mes} - r_{j, sim})^2}{2\sigma_{j, sim}^2} \right]$$

$$\sigma_{\text{Log } L}^2 \cong \sum_{j=1}^n \left( \frac{(r_{j, mes} - r_{j, sim})^2}{\sigma_{j, sim}^2} \right)^2 \times (\sigma_{j, mes}^2 + \sigma_{j, sim}^2)$$



# Task1: Simulated Reactor Library

| Reactor Type |
|--------------|
| PWR(2.35%)   |
| PWR(3.4%)    |
| PWR(4.45%)   |
| FBR          |
| PHWR         |
| NRX          |
| MAGNOX       |
| HFIR         |
| MURR         |

Reactor Model Burnup and Time Since Irradiation Ratio Set Matrix

Bu (0 → ~5 GWd/MTU)

0 d  
↓  
TSI  
↓  
5000 d

|  |  |  |  |  |  |  |  |  |  |  |
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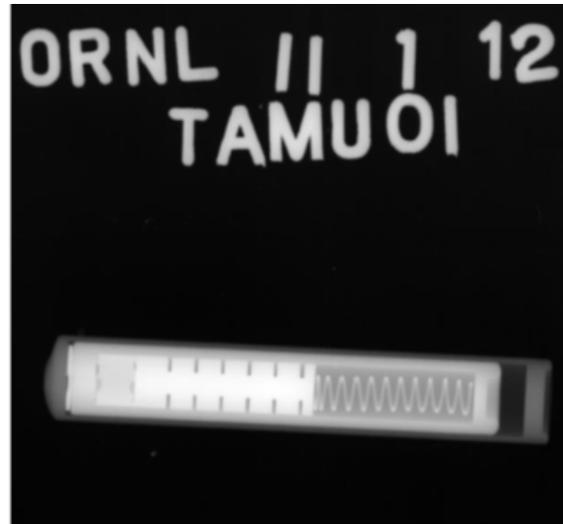
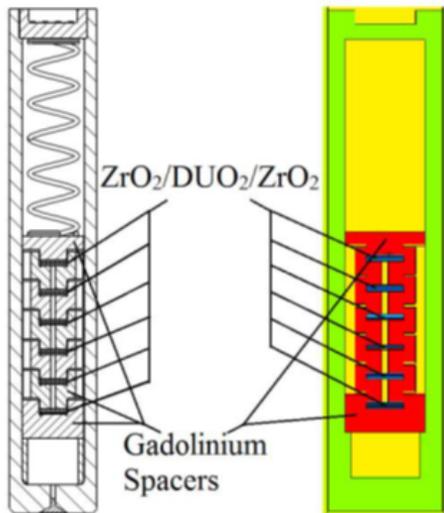
Ratio Set (Bu, TSI)

- $^{137}\text{Cs} / ^{133}\text{Cs}$
- $^{134}\text{Cs} / ^{137}\text{Cs}$
- $^{135}\text{Cs} / ^{137}\text{Cs}$
- $^{136}\text{Ba} / ^{138}\text{Ba}$
- $^{154}\text{Eu} / ^{153}\text{Eu}$
- $^{150}\text{Sm} / ^{149}\text{Sm}$
- $^{152}\text{Sm} / ^{149}\text{Sm}$
- $^{240}\text{Pu} / ^{239}\text{Pu}$
- $^{241}\text{Pu} / ^{239}\text{Pu}$
- $^{242}\text{Pu} / ^{239}\text{Pu}$

$$\text{Log } L(M, Bu, TSI | r_{mes}) = \sum_{j=1}^n \left[ \log \left( \frac{1}{\sigma_{j,sim} \sqrt{2\pi}} \right) - \frac{(r_{j,mes} - r_{j,sim})^2}{2\sigma_{j,sim}^2} \right]$$

# Task 2: Experimental Irradiation at HFIR

- Depleted UO<sub>2</sub> fuel samples irradiated at HFIR
- Gadolinium irradiation capsule
- Burnup = 4.36 ± 0.28 GWd/MTU
- Each pellet: 11 mg uranium, produced 200 µg plutonium with 87% <sup>239</sup>Pu
- TSI = 1601 days

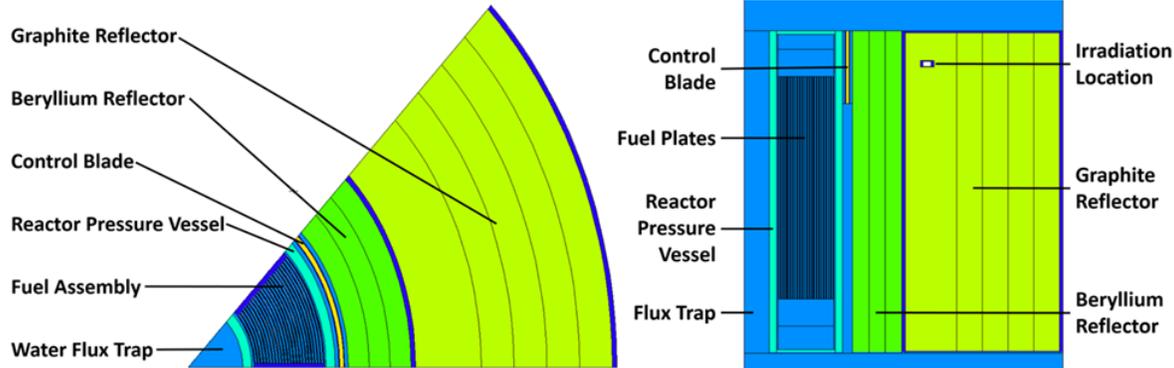


HFIR irradiated material mass spectroscopy measured set of intra-element ratio values

| Ratio                                | Measured Value          | Measurement Error |
|--------------------------------------|-------------------------|-------------------|
| <sup>137</sup> Cs/ <sup>133</sup> Cs | 1.30 × 10 <sup>0</sup>  | 6.7%              |
| <sup>134</sup> Cs/ <sup>137</sup> Cs | 3.74 × 10 <sup>-3</sup> | 4.2%              |
| <sup>135</sup> Cs/ <sup>137</sup> Cs | 4.25 × 10 <sup>-1</sup> | 10%               |
| <sup>154</sup> Eu/ <sup>153</sup> Eu | 4.67 × 10 <sup>-2</sup> | 4.5%              |
| <sup>150</sup> Sm/ <sup>149</sup> Sm | 3.23 × 10 <sup>0</sup>  | 2.7%              |
| <sup>152</sup> Sm/ <sup>149</sup> Sm | 2.93 × 10 <sup>0</sup>  | 1.3%              |
| <sup>240</sup> Pu/ <sup>239</sup> Pu | 8.28 × 10 <sup>-2</sup> | 0.59%             |
| <sup>241</sup> Pu/ <sup>239</sup> Pu | 3.30 × 10 <sup>-2</sup> | 0.88%             |
| <sup>242</sup> Pu/ <sup>239</sup> Pu | 1.88 × 10 <sup>-3</sup> | 0.88%             |

# Task 2: Experimental Irradiation at MURR

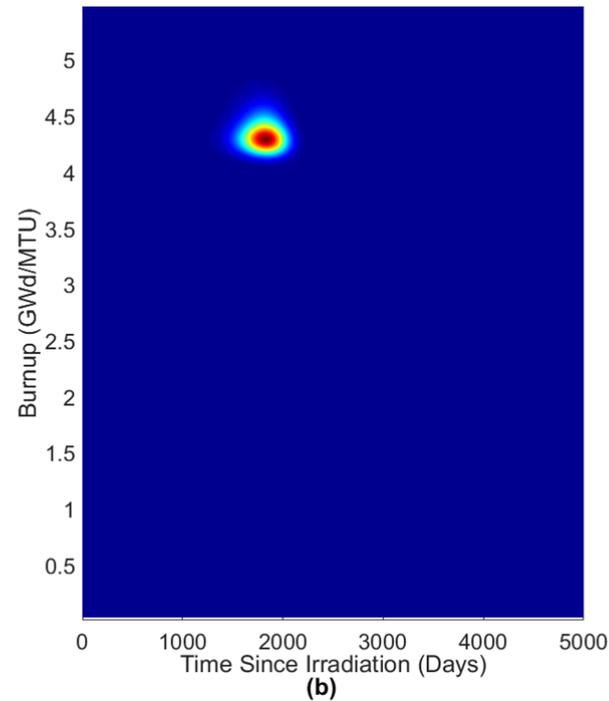
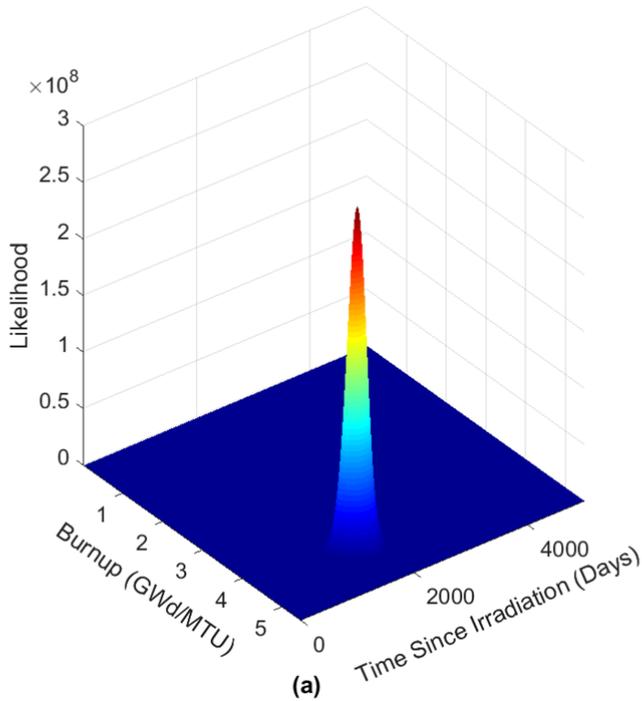
- Natural UO<sub>2</sub> fuel samples irradiated at MURR
- Complex irradiation history
- Irradiation history and axial/radial location of samples known
- Burnup =  $0.97 \pm 0.03$  GWd/MTU
- TSI = 318 days



MURR irradiated material mass spectroscopy measured set of intra-element ratio values

| Ratio                             | Measured Value        | Measurement Error |
|-----------------------------------|-----------------------|-------------------|
| $^{137}\text{Cs}/^{133}\text{Cs}$ | $9.75 \times 10^{-1}$ | 6.6%              |
| $^{134}\text{Cs}/^{137}\text{Cs}$ | $3.84 \times 10^{-3}$ | 7.0%              |
| $^{135}\text{Cs}/^{137}\text{Cs}$ | $2.95 \times 10^{-1}$ | 6.8%              |
| $^{150}\text{Sm}/^{149}\text{Sm}$ | $9.88 \times 10^0$    | 6.7%              |
| $^{152}\text{Sm}/^{149}\text{Sm}$ | $6.65 \times 10^0$    | 5.7%              |
| $^{240}\text{Pu}/^{239}\text{Pu}$ | $4.77 \times 10^{-2}$ | 5.7%              |
| $^{241}\text{Pu}/^{239}\text{Pu}$ | $2.40 \times 10^{-3}$ | 5.8%              |
| $^{242}\text{Pu}/^{239}\text{Pu}$ | $5.99 \times 10^{-5}$ | 8.3%              |

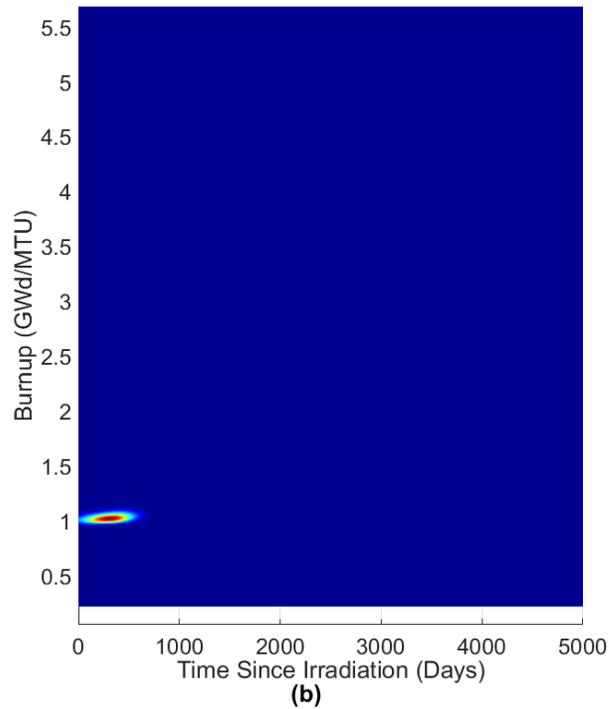
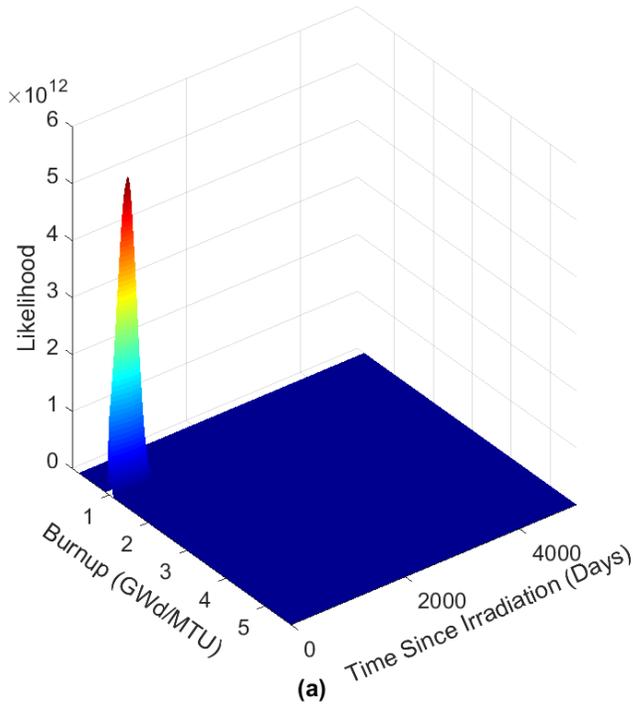
# Maximum Likelihood Results – HFIR Cont.



Results of Maximum Likelihood Analysis for the HFIR Irradiated Material (a) 3-D Likelihood Surface Map and (b) 2-D Contour Map for the Most Likely Reactor (HFIR)

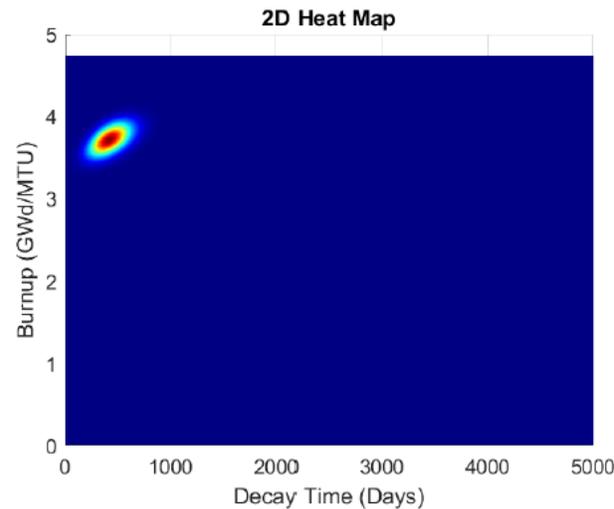
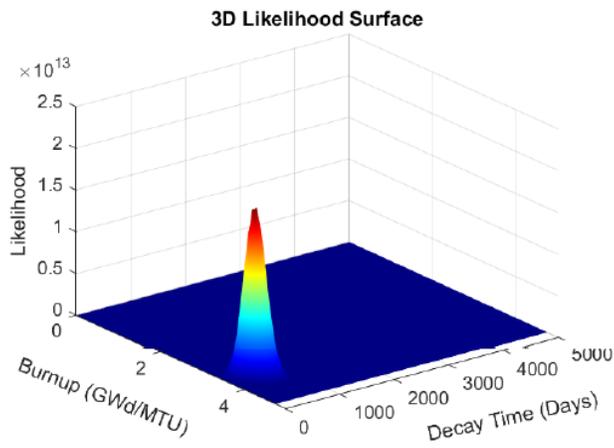
| Ratio                             | Measured Value        | Selected HFIR Simulation Value | Selected /Mes |
|-----------------------------------|-----------------------|--------------------------------|---------------|
| $^{137}\text{Cs}/^{133}\text{Cs}$ | $1.30 \times 10^0$    | $9.74 \times 10^{-1}$          | 0.73          |
| $^{134}\text{Cs}/^{137}\text{Cs}$ | $3.74 \times 10^{-3}$ | $3.71 \times 10^{-3}$          | 0.99          |
| $^{135}\text{Cs}/^{137}\text{Cs}$ | $4.25 \times 10^{-1}$ | $4.95 \times 10^{-1}$          | 1.16          |
| $^{154}\text{Eu}/^{153}\text{Eu}$ | $4.67 \times 10^{-2}$ | $4.66 \times 10^{-2}$          | 1.00          |
| $^{150}\text{Sm}/^{149}\text{Sm}$ | $3.23 \times 10^0$    | $3.33 \times 10^0$             | 1.03          |
| $^{152}\text{Sm}/^{149}\text{Sm}$ | $2.93 \times 10^0$    | $2.46 \times 10^0$             | 0.84          |
| $^{240}\text{Pu}/^{239}\text{Pu}$ | $8.28 \times 10^{-2}$ | $8.75 \times 10^{-2}$          | 1.06          |
| $^{241}\text{Pu}/^{239}\text{Pu}$ | $3.30 \times 10^{-2}$ | $4.22 \times 10^{-2}$          | 1.28          |
| $^{242}\text{Pu}/^{239}\text{Pu}$ | $1.88 \times 10^{-3}$ | $2.72 \times 10^{-3}$          | 1.44          |

# Maximum Likelihood Results – MURR Cont.



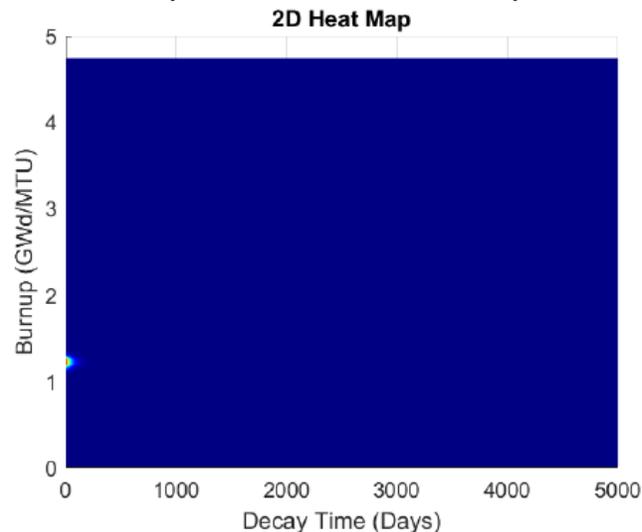
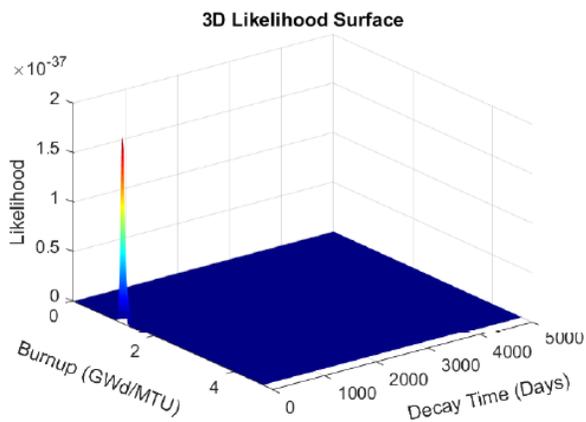
Results of Maximum Likelihood Analysis for the MURR Irradiated Material (a) 3-D Likelihood Surface Map and (b) 2-D Contour Map for the Most Likely Reactor (MURR)

| Ratio                             | Measured Value        | Selected MURR Simulation Value | Selected /Mes |
|-----------------------------------|-----------------------|--------------------------------|---------------|
| $^{137}\text{Cs}/^{133}\text{Cs}$ | $9.75 \times 10^{-1}$ | $9.43 \times 10^{-1}$          | 0.97          |
| $^{134}\text{Cs}/^{137}\text{Cs}$ | $3.84 \times 10^{-3}$ | $3.77 \times 10^{-3}$          | 0.98          |
| $^{135}\text{Cs}/^{137}\text{Cs}$ | $2.95 \times 10^{-1}$ | $2.77 \times 10^{-1}$          | 0.94          |
| $^{150}\text{Sm}/^{149}\text{Sm}$ | $9.88 \times 10^0$    | $1.02 \times 10^1$             | 1.03          |
| $^{152}\text{Sm}/^{149}\text{Sm}$ | $6.65 \times 10^0$    | $6.20 \times 10^0$             | 0.93          |
| $^{240}\text{Pu}/^{239}\text{Pu}$ | $4.77 \times 10^{-2}$ | $4.42 \times 10^{-2}$          | 0.93          |
| $^{241}\text{Pu}/^{239}\text{Pu}$ | $2.40 \times 10^{-3}$ | $2.30 \times 10^{-3}$          | 0.96          |
| $^{242}\text{Pu}/^{239}\text{Pu}$ | $5.99 \times 10^{-5}$ | $6.01 \times 10^{-5}$          | 1.00          |



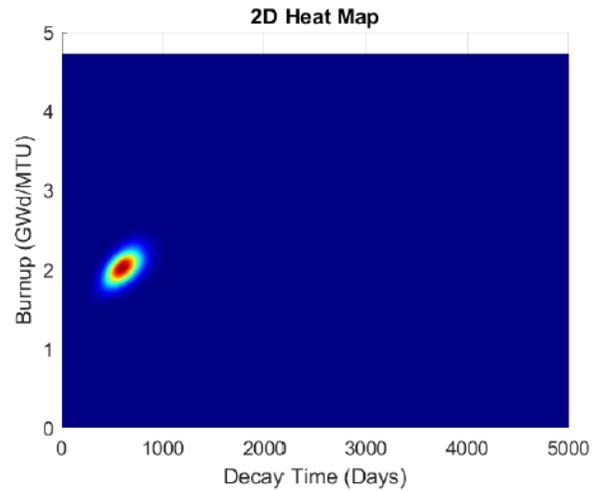
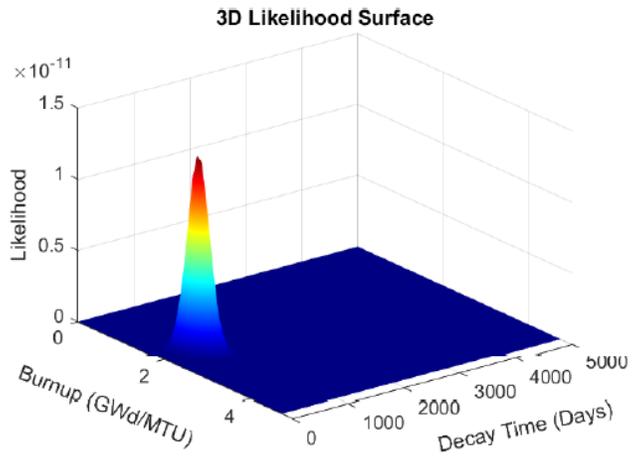
|      | Likelihood            | Burnup (GWD/MTU) | Time (days) |
|------|-----------------------|------------------|-------------|
| PWR  | $2.07 \times 10^{13}$ | 3.74             | 429         |
| PHWR | $3.44 \times 10^{-1}$ | 1.28             | 1           |
| FBR  | 0                     |                  |             |

Spoofer 1: 50% PWR (4 GWD/MTU) and 50% PHWR (1 GWD/MTU) with one year cooling period



|      | Likelihood             | Burnup (GWD/MTU) | Time (days) |
|------|------------------------|------------------|-------------|
| PWR  | $1.88 \times 10^{-37}$ | 1.2              | 1           |
| PHWR | $1.44 \times 10^{-67}$ | 0.35             | 1           |
| FBR  | $4.14 \times 10^{-69}$ | 0.92             | 1           |

Spoofer 2: 50% PWR (4 GWD/MTU) and 50% FBR (2 GWD/MTU) with one year cooling period

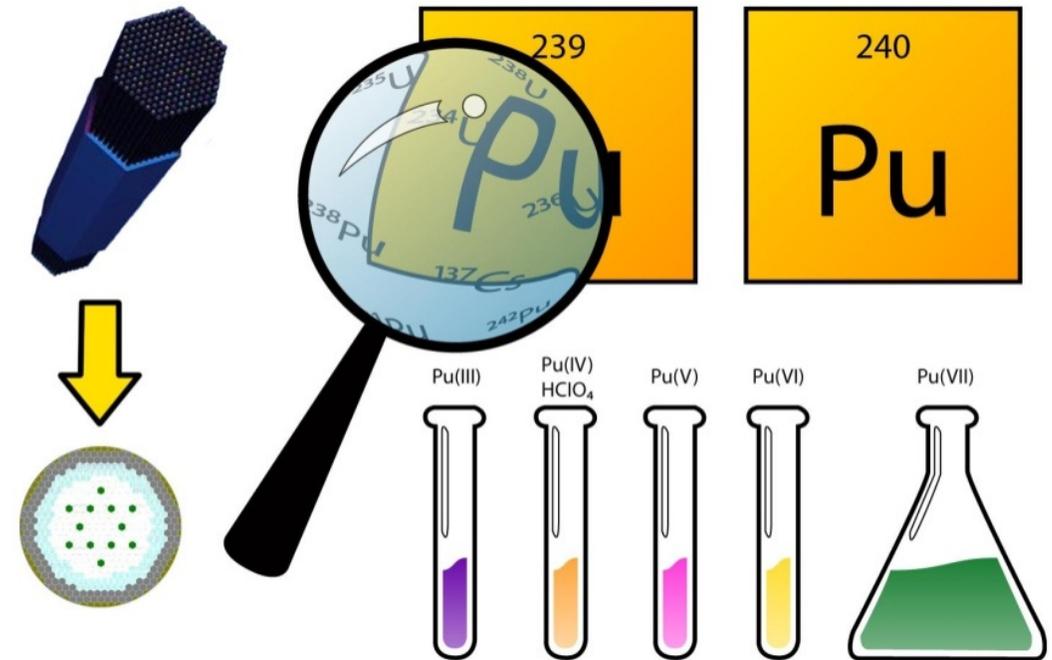


|      | Likelihood             | Burnup (GWd/MTU) | Time (days) |
|------|------------------------|------------------|-------------|
| PWR  | $9.44 \times 10^{-75}$ | 0.54             | 1           |
| PHWR | $4.60 \times 10^{-92}$ | 0.14             | 1           |
| FBR  | $1.38 \times 10^{-11}$ | 2.01             | 592         |

Spooof 3: 50% PHWR (1 GWD/MTU) and 50% FBR (2 GWD/MTU) with one year cooling period

# Expected Impact

- Successful completion of this project will enhance the US capability to monitor FNFCs
  - Application in environmental and wide area environmental sample analyses (Safeguards monitoring)
  - Application in nuclear security (Material out of regulatory control)
- A library for various combinations of FNFC facility operations that could produce Pu and  $^{233}\text{U}$



# MTV Impact

- Two PhD students and one MS student
- Internships in LLNL and ANL and potential transitions as employees
- Faculty time in LLNL and ANL
- Workshops:
  - Reactor core physics simulation with MCNP6 for users with single user license
  - Radiochemistry analytical techniques
- Technology transitions
  - National lab collaborations and potential transition of the methodology
  - Collaborated with DHS in the past on similar project



# Conclusion

- This project will enhance US capability to monitor FNFC
- We will produce a library for various combinations of FNFC facility operations that could produce Pu and  $^{233}\text{U}$ . The data will be used to inform and validate specific facility models
- Nuclear engineering students with nuclear reactor core and fuel cycle modeling, simulation and radiochemical expertise for potential transition to national laboratories
- Workshops: To enhance MTV students/researcher expertise



# Next Steps

- New experimental irradiation of LEU in MURR and analysis
- Fuel separation modeling in collaboration with ANL and LLNL
- Thorium irradiation modeling, separation and analysis



# Acknowledgements



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# Backup Slides

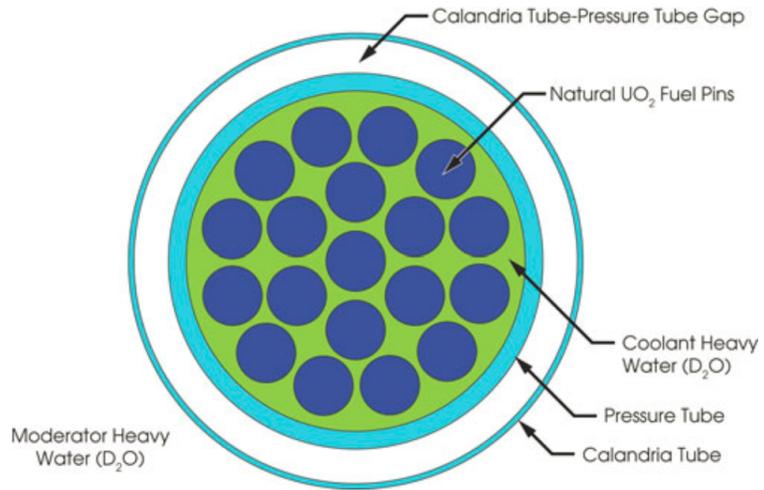


# Reactor Library Model Characteristics

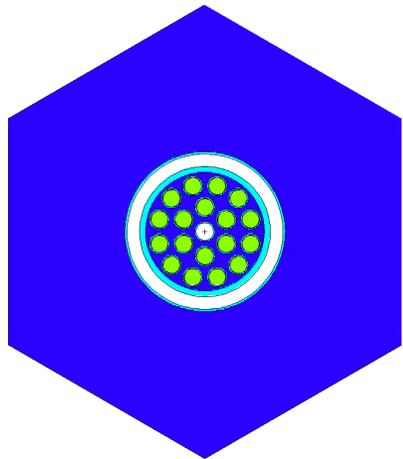
| Reactor Model      | Power (MWth) | Fuel Type (at.% <sup>235</sup> U) | Moderator   | Coolant        |
|--------------------|--------------|-----------------------------------|-------------|----------------|
| PWR (2.35%)        | 3400         | UO <sub>2</sub> (2.35)            | Light Water | Light Water    |
| PWR (3.4%)         | 3400         | UO <sub>2</sub> (3.4)             | Light Water | Light Water    |
| PWR (4.45%)        | 3400         | UO <sub>2</sub> (4.45)            | Light Water | Light Water    |
| FBR (blanket)      | 1250         | UO <sub>2</sub> (0.25)            | -           | Liquid Sodium  |
| PHWR               | 756          | UO <sub>2</sub> (0.72)            | Heavy Water | Heavy Water    |
| NRX                | 40           | UO <sub>2</sub> (0.72)            | Heavy Water | Heavy Water    |
| MAGNOX             | 25           | U metal w/ 0.5% Al (0.72)         | Graphite    | Carbon Dioxide |
| HFIR (irradiation) | 85           | UO <sub>2</sub> (0.25)            | Light Water | Light Water    |
| MURR (irradiation) | 10           | UO <sub>2</sub> (0.72)            | Light Water | Light Water    |



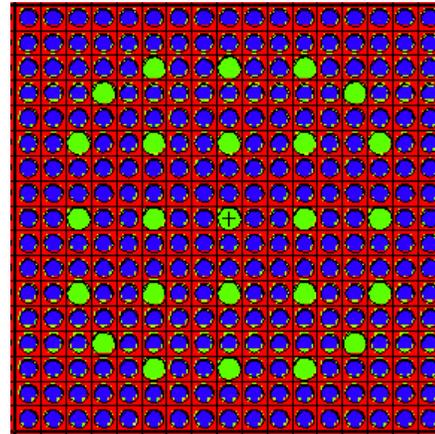
# MCNP Reactor Models



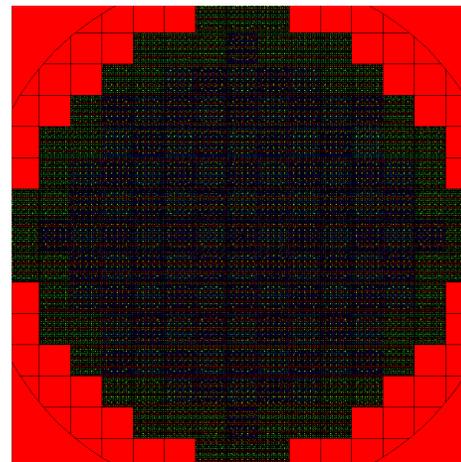
PHWR



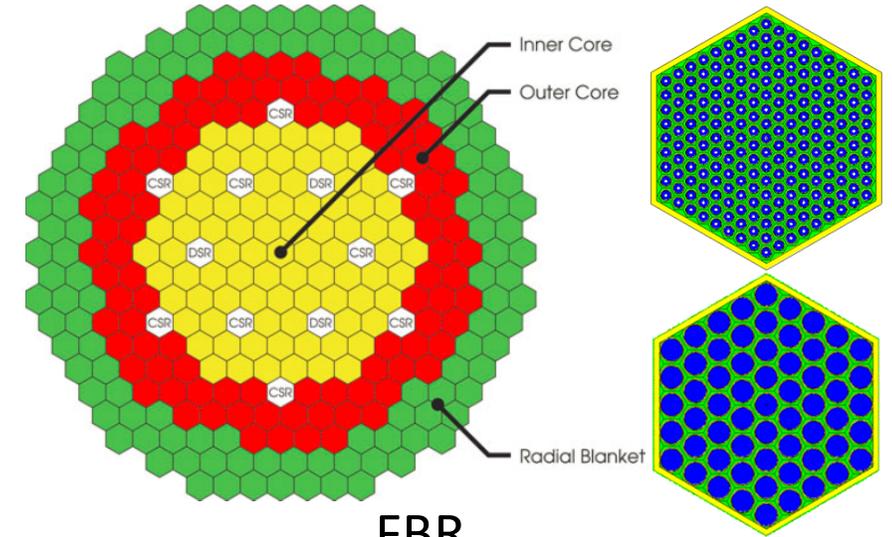
NRX



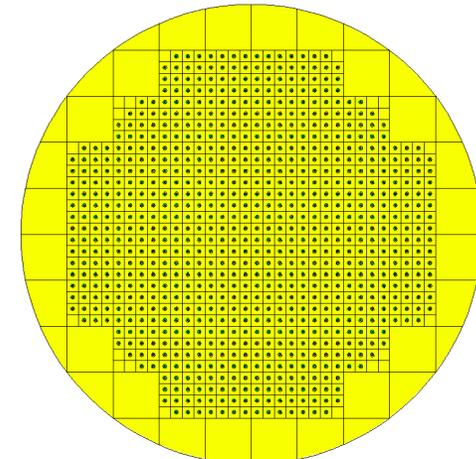
PWR Assembly



PWR

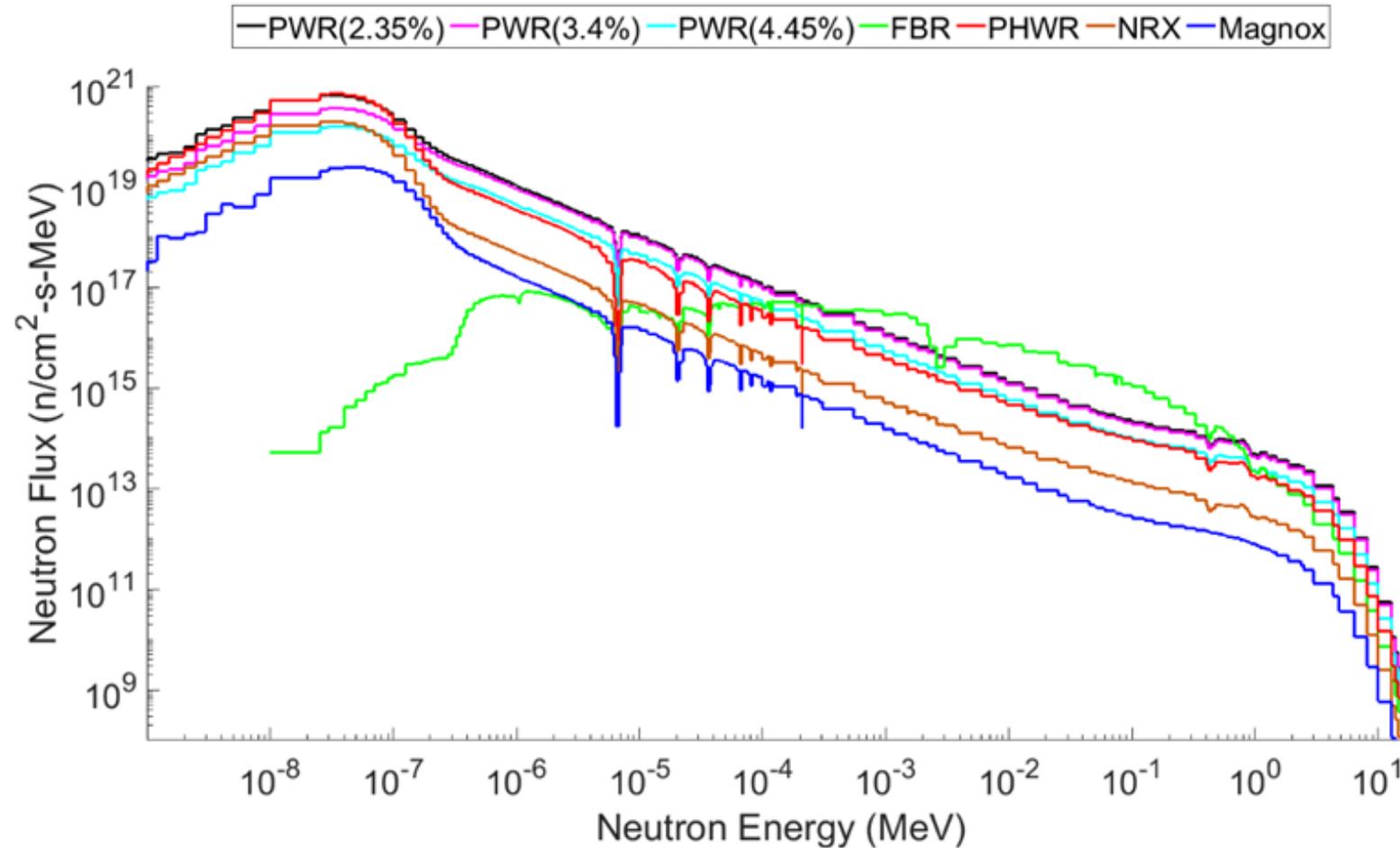


FBR

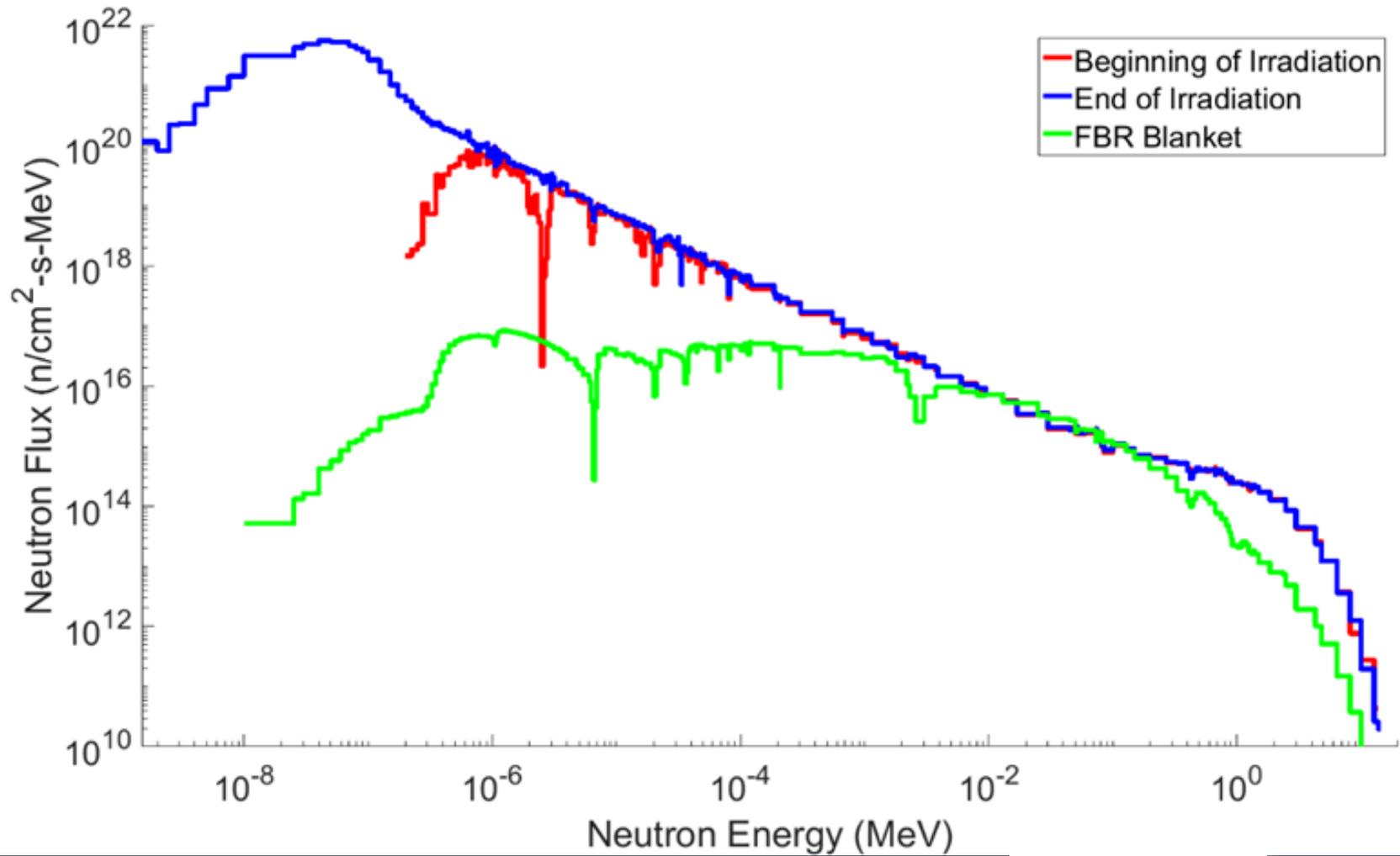


Magnox

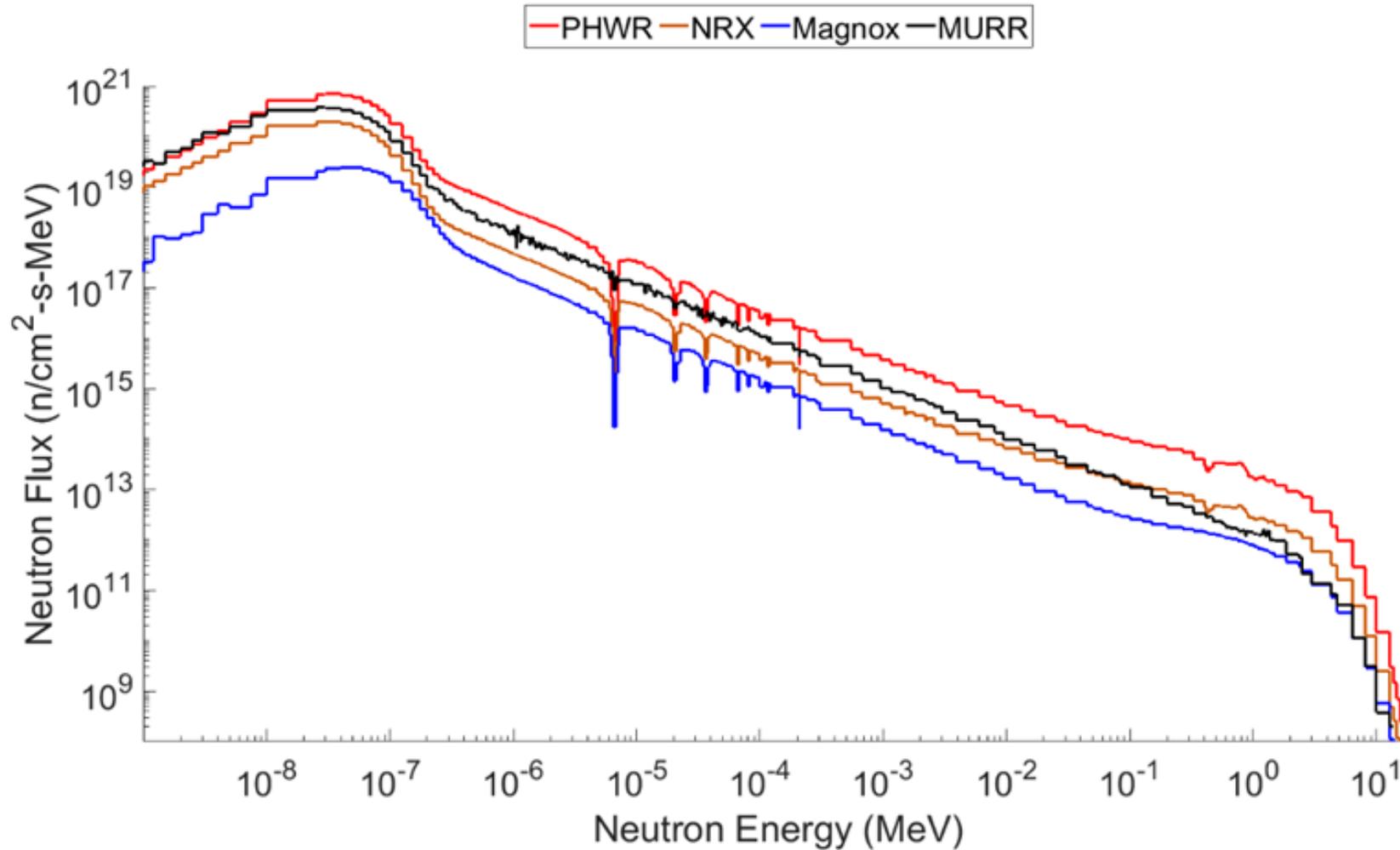
# Reactor Library Neutron Spectra



# HFIR Irradiation Spectrum



# MURR Irradiation Spectrum



# MURR Irradiation Burnup

- Determination of burnup via  $^{137}\text{Cs}$  measurements

- $$A_n = \frac{CPS_n}{\epsilon_\gamma * Y_\gamma}$$

| Fuel Disc | CPS (Live) | Measured $^{137}\text{Cs}$ Activity (Bq) | End of Irradiation $^{137}\text{Cs}$ Activity (Bq) |
|-----------|------------|--|--|
| A         | 69.86      | $1.65 \times 10^6 \pm 2.48 \times 10^4$  | $1.67 \times 10^6 \pm 2.51 \times 10^4$            |
| B         | 70.25      | $1.66 \times 10^6 \pm 2.50 \times 10^4$  | $1.68 \times 10^6 \pm 2.52 \times 10^4$            |
| C         | 71.66      | $1.70 \times 10^6 \pm 2.55 \times 10^4$  | $1.71 \times 10^6 \pm 2.57 \times 10^4$            |



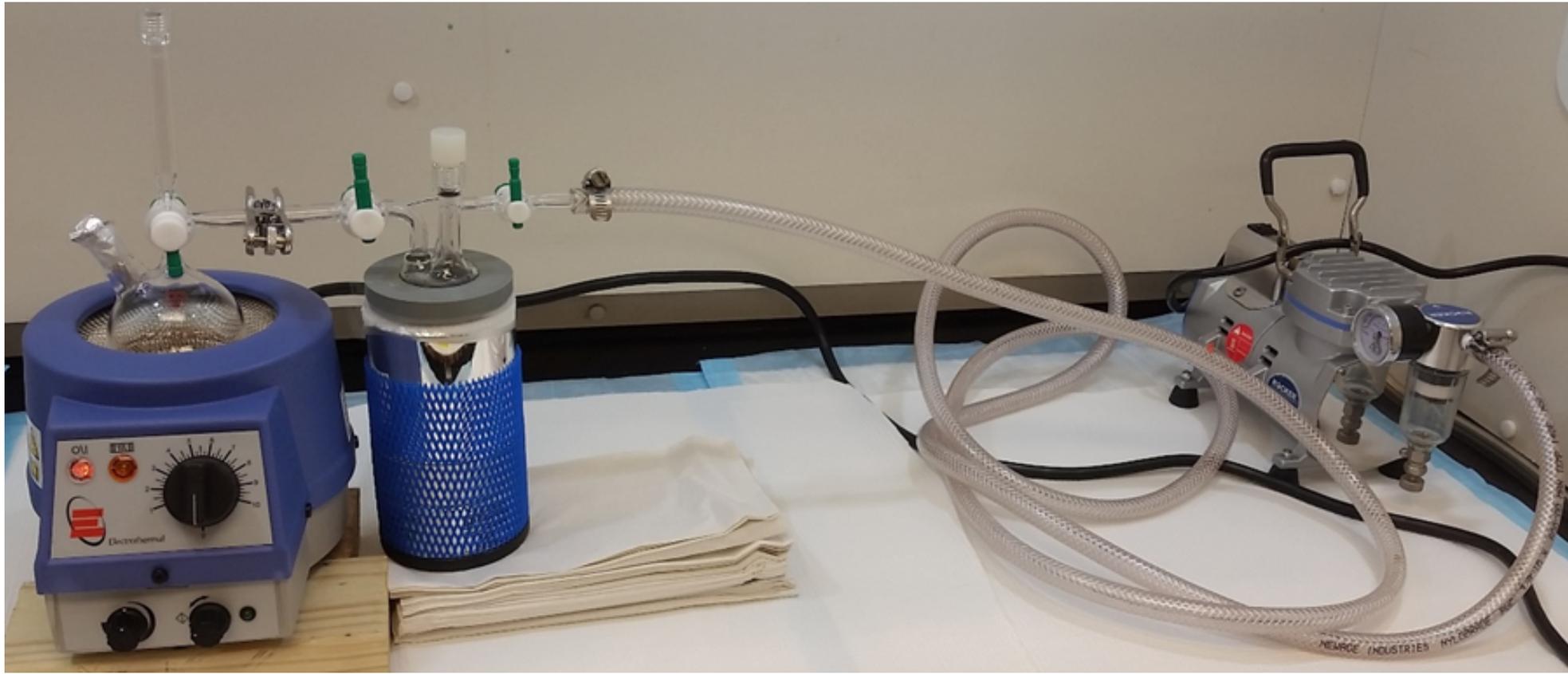
# MURR Irradiation Burnup Cont.

- $Bu = \frac{N_{137Cs} * Q * GWd}{Y_{137} * U}$ 
  - $Q = 202 \pm 5$  MeV
  - $GWd = (1 \text{ GWd} = 5.393 \times 10^{26} \text{ MeV})$
  - $Y_{137} = 6.221\% \pm 0.069\%$
  - $U = 14.52 \text{ mg} \pm 0.23 \text{ mg} (1.452 \times 10^{-9} \text{ MT})$

| Fuel Disc | Measured Burnup (GWd/MTU) | Measured Burnup Error | Simulated Burnup (GWd/MTU) | Simulated Burnup Error | S/E         |
|-----------|---------------------------|-----------------------|----------------------------|------------------------|-------------|
| A         | 0.949                     | 3.32%                 | 0.960                      | 0.086%                 | 1.01 ± 0.03 |
| B         | 0.954                     | 3.32%                 | 0.960                      | 0.086%                 | 1.01 ± 0.03 |
| C         | 0.973                     | 3.32%                 | 0.960                      | 0.086%                 | 0.99 ± 0.03 |



# Irradiated Sample Dissolution Setup



# Irradiated Sample Measurements

- Gamma spectrometry measurements using a Canberra HPGe
- Mass spectrometry measurements using a Thermo Fisher Scientific ICP-MS



# MURR Gamma Spectrometry

| Isotope           | Measured Activity in Full Disc (Bq) | Count Rate (CPS) Error | Simulated Activity in Full Disc (Bq) | S/E  |
|-------------------|-------------------------------------|------------------------|--------------------------------------|------|
| $^{95}\text{Zr}$  | $2.54 \times 10^7$                  | < 0.1%                 | $2.49 \times 10^7$                   | 0.98 |
| $^{103}\text{Ru}$ | $4.92 \times 10^6$                  | 0.3%                   | $5.62 \times 10^6$                   | 1.14 |
| $^{134}\text{Cs}$ | $1.12 \times 10^5$                  | 2.2%                   | $9.92 \times 10^4$                   | 0.89 |
| $^{137}\text{Cs}$ | $1.71 \times 10^6$                  | 0.1%                   | $1.67 \times 10^6$                   | 0.98 |
| $^{141}\text{Ce}$ | $4.21 \times 10^6$                  | 0.1%                   | $4.77 \times 10^6$                   | 1.13 |
| $^{144}\text{Ce}$ | $2.94 \times 10^7$                  | < 0.1%                 | $3.11 \times 10^7$                   | 1.06 |



# MURR Mass Spectrometry

| Isotope           | Fissiogenic Ratio | Measured Mass (g)     | Measured Mass Relative Error | Simulated Mass (g)    | S/E  |
|-------------------|-------------------|-----------------------|------------------------------|-----------------------|------|
| $^{133}\text{Cs}$ | 1                 | $5.22 \times 10^{-7}$ | 6.0%                         | $5.42 \times 10^{-7}$ | 1.04 |
| $^{135}\text{Cs}$ | 1                 | $1.50 \times 10^{-7}$ | 6.2%                         | $1.42 \times 10^{-7}$ | 0.94 |
| $^{137}\text{Cs}$ | 0.976             | $5.08 \times 10^{-7}$ | 6.0%                         | $5.14 \times 10^{-7}$ | 1.01 |
| $^{148}\text{Nd}$ | 0.983             | $1.55 \times 10^{-7}$ | 5.8%                         | $1.54 \times 10^{-7}$ | 0.99 |
| $^{149}\text{Sm}$ | 1                 | $8.34 \times 10^{-9}$ | 5.8%                         | $9.51 \times 10^{-9}$ | 1.14 |
| $^{150}\text{Sm}$ | 0.589             | $9.22 \times 10^{-8}$ | 5.8%                         | $9.24 \times 10^{-8}$ | 1.00 |
| $^{152}\text{Sm}$ | 1                 | $5.55 \times 10^{-8}$ | 5.8%                         | $5.58 \times 10^{-8}$ | 1.01 |
| $^{153}\text{Eu}$ | 1                 | $1.76 \times 10^{-8}$ | 5.9%                         | $1.87 \times 10^{-8}$ | 1.06 |



# MURR Mass Spectrometry - Plutonium

| Measured Pu Mass ( $\mu\text{g}$ ) | Measurement Error | Simulated Pu Mass ( $\mu\text{g}$ ) | Simulated Stochastic Error | S/E             |
|------------------------------------|-------------------|-------------------------------------|----------------------------|-----------------|
| 20.1                               | 5.3%              | 20.9                                | 0.78%                      | $1.04 \pm 0.06$ |

| Isotope           | Measured Pu Vector | Measured Pu Vector Relative Error | Simulated Pu Vector | S/E <sup>a</sup> |
|-------------------|--------------------|-----------------------------------|---------------------|------------------|
| $^{239}\text{Pu}$ | 95.22%             | 0.1%                              | 95.75%              | 1.01             |
| $^{240}\text{Pu}$ | 4.55%              | 2.2%                              | 4.05%               | 0.89             |
| $^{241}\text{Pu}$ | 0.23%              | 1.9%                              | 0.20%               | 0.86             |
| $^{242}\text{Pu}$ | <0.01%             | N/A                               | <0.01%              | N/A              |



# Maximum Likelihood Calculation

- Likelihood equation:

$$L(M, Bu, TSI|r_{mes}) \propto f(r_{mes}|M, Bu, TSI) = \prod_{j=1}^n \frac{1}{\sigma_{j,sim}\sqrt{2\pi}} \exp\left\{-\frac{(r_{j,mes}-r_{j,sim})^2}{2\sigma_{j,sim}^2}\right\}$$

- Log-likelihood equation:

$$\text{Log } L(M, Bu, TSI|r_{mes}) = \sum_{j=1}^n \left[ \log\left(\frac{1}{\sigma_{j,sim}\sqrt{2\pi}}\right) - \frac{(r_{j,mes}-r_{j,sim})^2}{2\sigma_{j,sim}^2} \right]$$

- Variance in the log-likelihood:

$$\sigma_{\text{Log } L}^2 = \sum_{j=1}^n \left( \frac{(r_{j,mes}-r_{j,sim})}{\sigma_{j,sim}} \right)^2 \times (\sigma_{j,mes}^2 + \sigma_{j,sim}^2)$$

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# Maximum Likelihood Results – MURR

Results of the Maximum Likelihood Analysis for the MURR Irradiated Material <sup>a</sup>

| Reactor Model | Log-Likelihood Value <sup>b</sup>                | Predicted Burnup<br>(GWd/MTU) | Predicted Time Since<br>Irradiation (days) |
|---------------|--|-------------------------------|--|
| MURR          | +29.5 ± 1.1                                      | 1.02                          | 295  |
| NRX           | +25.3 ± 3.0                                      | 1.03                          | 208  |
| MAGNOX        | +13.2 ± 5.7                                      | 0.73                          | 0  |
| PWR (3.4%)    | -6.02 ± 8.71                                     | 3.91                          | 1381                                       |
| PWR (4.45%)   | -8.88 ± 10.2                                     | ≥ 3.90                        | 1196                                       |
| PWR (2.35%)   | -12.7 ± 10.2                                     | 3.10                          | 1202                                       |
| PHWR          | -14.7 ± 13.8                                     | 1.02                          | 360  |
| HFIR          | -166 ± 28  | 4.40                          | 1790                                       |
| FBR           | -1.52 × 10 <sup>5</sup> ± 2.02 × 10 <sup>4</sup> | ≥ 4.73                        | 0  |

<sup>a</sup> True Bu = 0.97 ± 0.03 GWd/MTU, True TSI = 318 days

<sup>b</sup> Maximum possible log-likelihood is 29.7



# Task 3: Maximum Likelihood Results – HFIR

Results of the Maximum Likelihood Analysis for the HFIR Irradiated Material <sup>a</sup>

| Reactor Model | Log-Likelihood Value <sup>b</sup>                | Predicted Burnup<br>(GWd/MTU) | Predicted Time Since<br>Irradiation (days) |
|---------------|--|-------------------------------|--|
| HFIR          | +19.5 ± 4.6                                      | 4.29                          | 1827                                       |
| MURR          | -46.6 ± 12.8                                     | 4.16                          | 1700                                       |
| NRX           | -52.5 ± 12.5                                     | 4.13                          | 1590                                       |
| MAGNOX        | -59.5 ± 13.3                                     | 3.00                          | 421  |
| PWR (2.35%)   | -86.7 ± 21.0                                     | ≥ 5.31                        | 1705                                       |
| PHWR          | -129 ± 32  | ≥ 4.35                        | 2308                                       |
| PWR (3.4%)    | -284 ± 26  | ≥ 5.01                        | 0  |
| PWR (4.45%)   | -5.27 × 10 <sup>3</sup> ± 1.38 × 10 <sup>2</sup> | ≥ 3.90                        | 0  |
| FBR           | -6.39 × 10 <sup>5</sup> ± 1.05 × 10 <sup>4</sup> | ≥ 4.73                        | 0  |

<sup>a</sup> True Bu = 4.36 ± 0.28 GWd/MTU, True TSI = 1601 days

<sup>b</sup> Maximum possible log-likelihood is 28.5

