Preliminary Validation of the Two Region Point Kinetics Model for Rossi-alpha Measurements

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Introduction and Motivation

- Effective multiplication factor, k_{eff}, of fissionable assemblies is crucial to nuclear nonproliferation, safeguards, and criticality safety
- k_{eff} is the ratio of neutrons in the $(n^{th}+1)$ over the n^{th} generations of a fissionable assembly
- Rossi-alpha measurements estimate the prompt neutron decay constant, α, to infer k_{eff}, which cannot be measured directly
- Prior work by Hua et al. showed the one-region point kinetics model is skewed by varying core-to-detector lifetimes, l_{ctd}, of reflected assemblies detected by ^3He gas proportional counter based systems, but two-region point kinetics model was not skewed by varying l_{ctd}.
- Next step is to investigate the two-region point kinetics model for varying reflector types and amounts with organic scintillator detection systems

Rossi-alpha

- Neutron emission times from fission events are nonrandom and time separation between neutron detections can be analyzed to characterize nuclear material
- Prompt neutron decay constant, α, determined from fitting of time difference histogram
- Reactivity, ρ, and effective multiplication factor, k_{eff}, cannot be measured but are related to α with: $\rho = \frac{k_{eff}-1}{k_{eff}} = \beta_{eff} - \alpha\Lambda$.
- Can estimate ρ and k_{eff} given a measured α by determining β_{eff} and Λ, typically by simulation

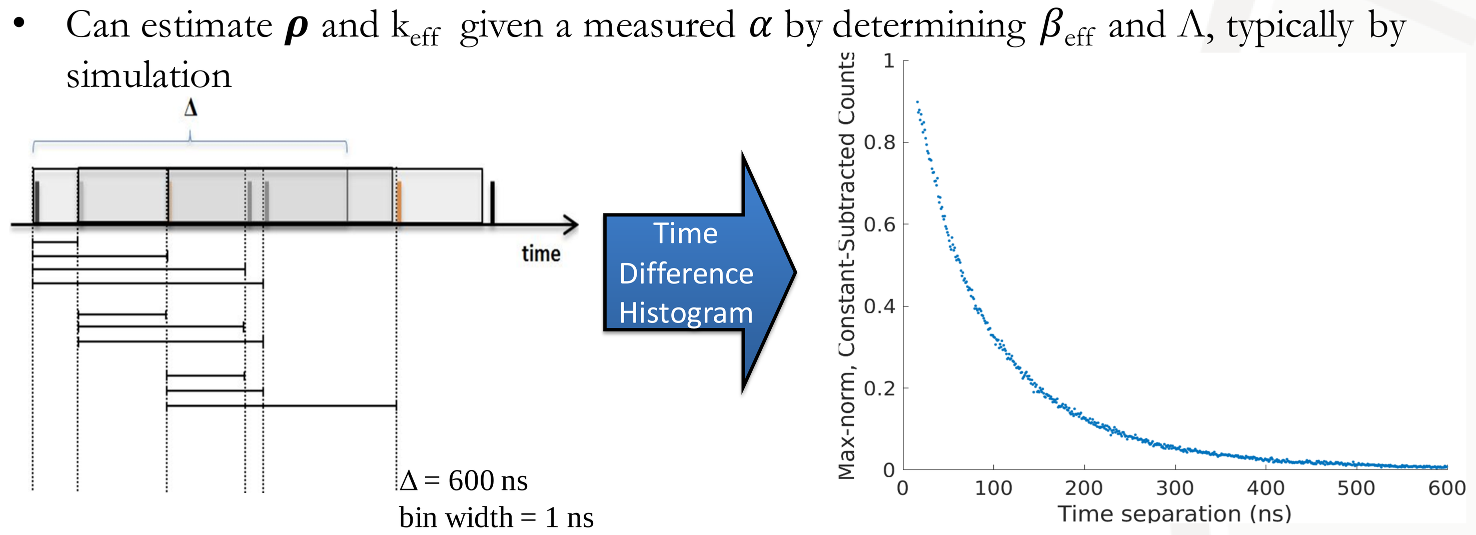
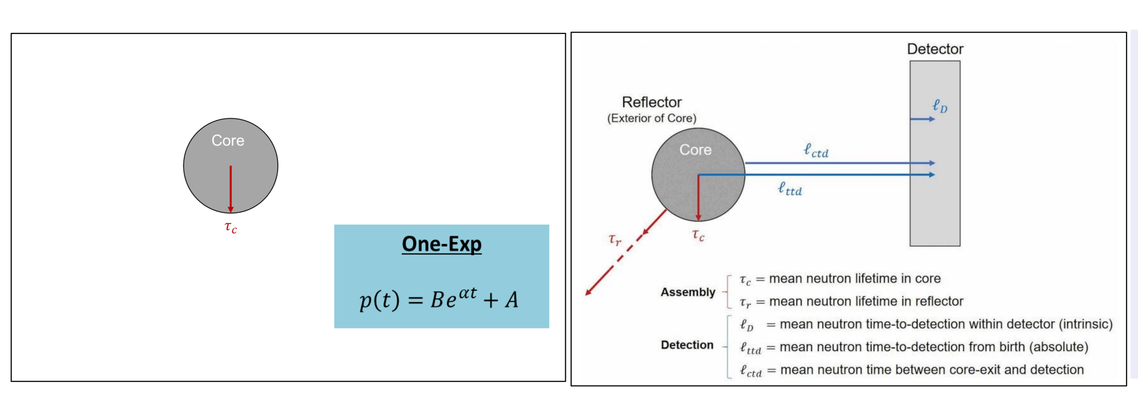


Fig. 1. (Left) Type-I binning time difference become a (right) Rossi-alpha histogram

One-Region vs. Two-Region Models

- Traditional Rossi-alpha only considers a fissile core (one-exponential model), which omits reflector and detector considerations
- Previous works statistically show that a two-exp model is a more adequate fit than the one-exponential model for reflected assemblies, but not how to get α from fit
- Developed a two-region point-kinetic model (and thus a two-exponential model) for reflected assemblies



Two-Exp

$$p(t) = A - C\left[e^{r_1 t}\rho_1 + e^{r_2 t}\rho_2\right]$$

$$\rho_1 = \frac{(1-R)^2}{r_1} + \frac{2(1-R)(R)}{r_1+r_2}, \quad \text{and}$$
$$\rho_2 = \frac{R^2}{r_2} + \frac{2(1-R)(R)}{r_1+r_2}.$$

$$\alpha = r_1(1-R) + r_2 R. \qquad \ell_{ctd} = -\frac{1}{r_2(1-R)+r_1 R}$$

Fig. 2. (Left) One-exp model that considers the mean neutron time in the core as opposed to (Center) two-exp model that includes variable core-to-detector lifetime, l_{ctd}, described by (Right) two-exp system of equations.

Experimental Setup

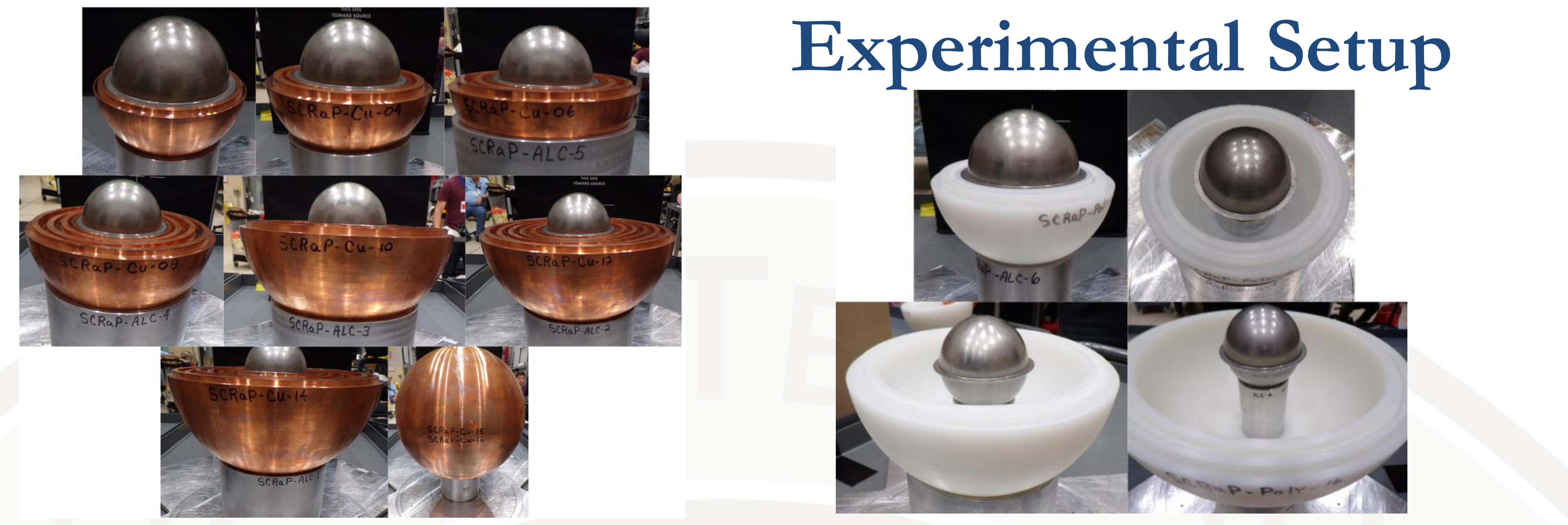


Fig. 3. (Left) Experimental setup of 0.5:0.5:4.0 inch copper shells surrounding BeRP ball and (Right) one inch polyethylene (PE) shells with inner offsets of 0, 1, 2, and 3 inches from the BeRP ball.

Measurement and Analysis

- Figure 3 is based on previous Device Assembly Facility (DAF) measurements
- Analyzing measurements from two 12 trans-stilbene scintillator arrays
- 4.5 kg of alpha-phase, beryllium reflected weapons-grade plutonium (BeRP ball) in two types of varying reflector configurations
  - ➢ Copper: 0.5 to 4.0 inch copper reflector in 0.5 inch increments
  - ➢ Polyethylene (PE): 1 inch PE shield with varying inner offset of 0, 1, 2, & 3 inches

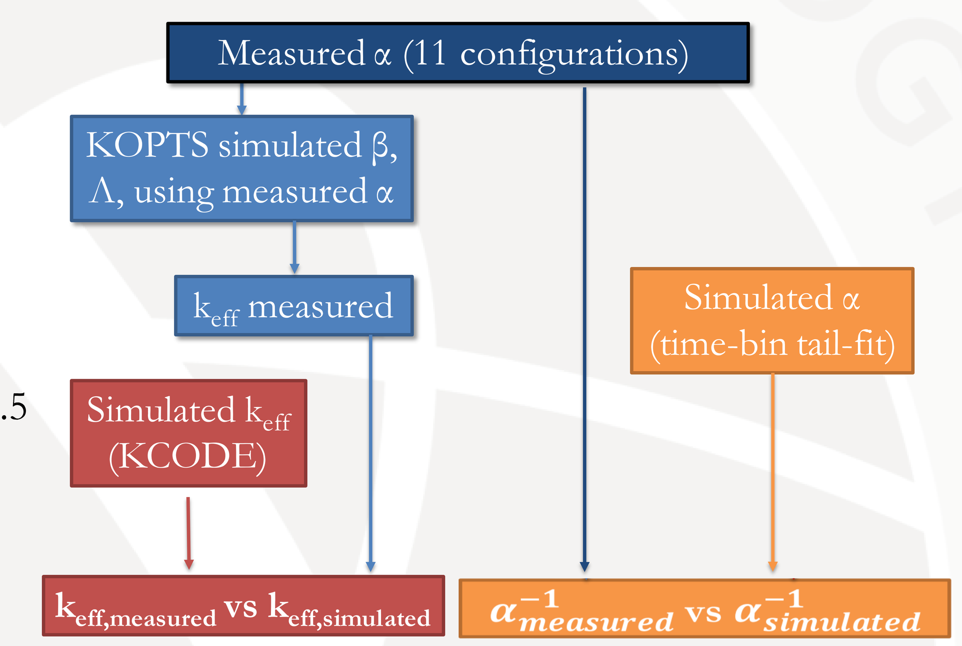


Fig. 4. Process of independently simulating and comparing time-bin tail-fit and KCODE.

Results

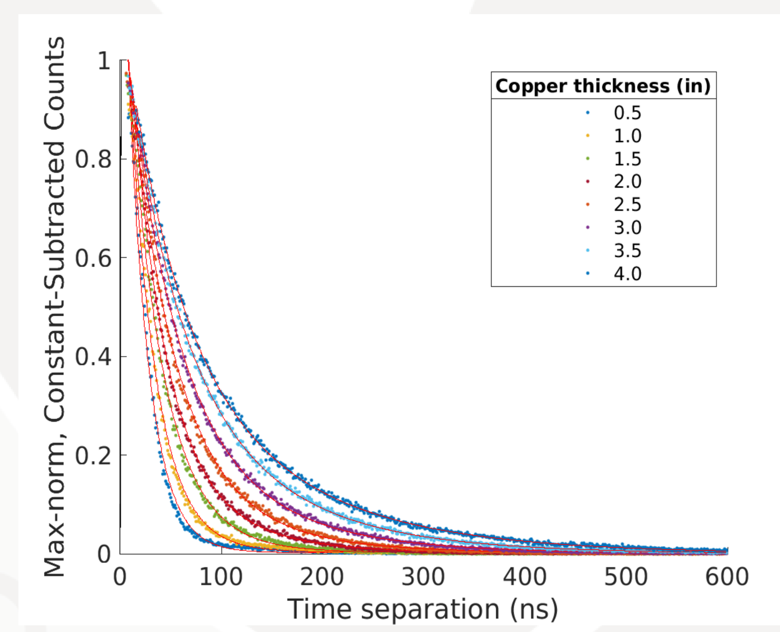


Fig. 5. Peak normalized Rossi-alpha histograms for each copper thickness with fit lines

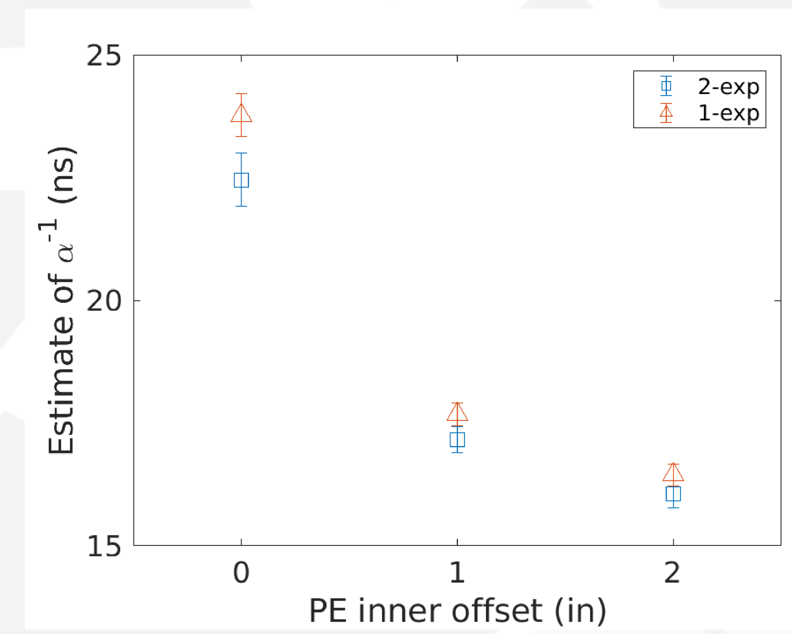
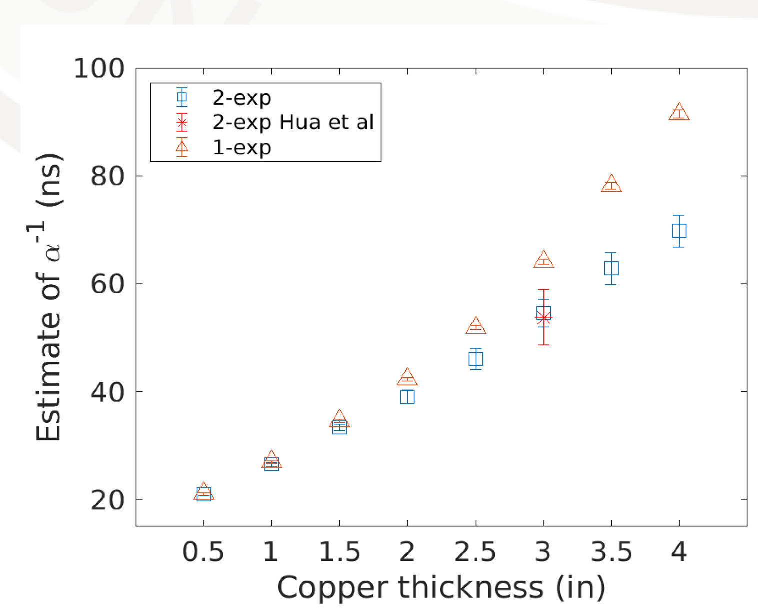


Fig. 6. α^{-1} estimates for each polyethylene offset except the three inch case



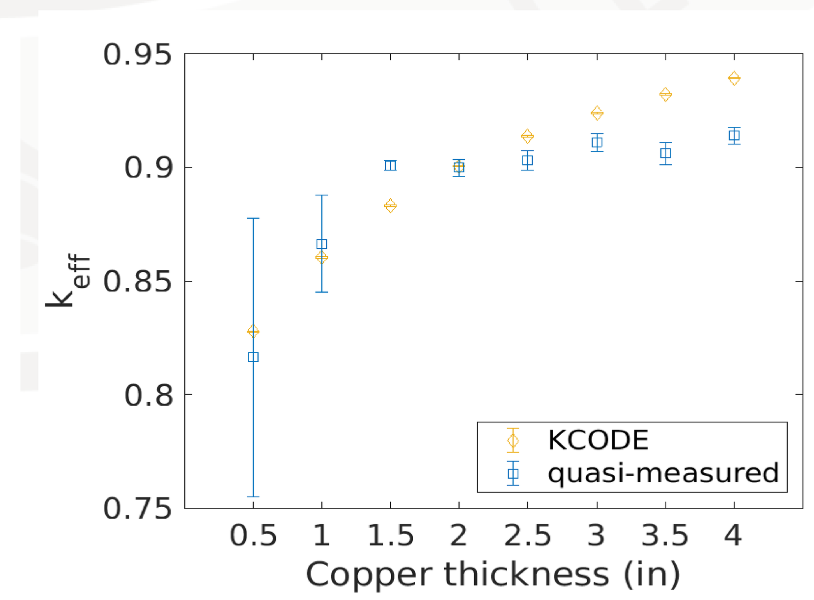


Fig. 7. (Left) α^{-1} estimates for each copper thickness and subsequent (Right) comparison of KCODE $k_{eff\text{-}simulated}$ and $k_{eff\text{-}measured}$ for the two-exponential model

Results Discussion

- Array of organic scintillators was used to measure 11 subcritical assemblies: a sphere of 4.5 kg of alpha-phase, weapons grade plutonium reflected by 0.5:0.5:4.0 inches of copper and 1 inch of PE with varying inner offsets of 0, 1, 2, and 3 inches
- Aim to use simulation to validate use of two region model for reflected assemblies and compare to the one region model
- Figure 4 (left) α^{-1} increases with increasing copper reflector with the one-region method consistently yielding higher estimates of α^{-1} than the two-region method
- Figure 3 (right) α^{-1} decreases with increasing polyethylene inner offset with the one-region model consistently yielding higher estimates of α^{-1} than the two-region method
- Figure 4 (right) is the result of using measured α and simulated values of β_{eff} and Λ to calculate $k_{eff\text{-}measured}$ and compare to $k_{eff\text{-}simulated}$ for varying copper-reflected thickness
  - ➢ Measured k_{eff} values are greater than simulated k_{eff} values for 1.0 & 1.5 inch reflectors, nearly the same for 2.0 inch reflector, and all less for 0.5 & 2.5-4.0 inch reflectors
  - ➢ Error and uncertainty in $k_{eff\text{-}measured}$ increases as $k_{eff\text{-}measured}$ decreases since the model assumes $k_{eff} \sim 1$, thus performing worse for more subcritical systems
  - ➢ Error increases drastically for reflectors less than 1.5 inch

Preliminary Conclusions

- One-region estimates of α^{-1} are skewed toward higher estimates of α^{-1} since this method cannot deconvolve l_{ctd}
- Greater discrepancy between one region and two region methods as reflector thickness increases
  - ➢ Two region method is more accurate as reflector thickness increases as expected
- KOPTS is producing unusual error for Λ for copper-reflected thicknesses 1.5 inches

Future Work

- Refine measurements for 3 inch PE inner offset for analysis
- Improve KOPTS simulations for copper to reduce error for Λ
- Simulate $\alpha_{simulated}$ for the copper cases
- Simulate polyethylene cases for KCODE, KOPTS, and $\alpha_{simulated}$ values once copper cases have been refined
- Begin investigating two-exp parameter changes between configurations

Impact

- **Expected:** Increase accuracy of Rossi-alpha method for reflected assemblies of fissionable material
- **MTV:** Prepare for MTV summer fellow work with DNNG

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